Multiple Criteria Decision Analysis Techniques in Aircraft Design and Evaluation Processes

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Abstract

The competitiveness of an aircraft is no longer dominated by economic criteria. In addition to the economic consideration, there are several other criteria needed to be taken into account in aircraft design and evaluation decision making processes. For instance, environmental aspects and level of comfort. Therefore, considering these multiple criteria simultaneously, aircraft design and aircraft evaluation are typical multi-criteria decision making problems.

Applying Multi-Criteria Decision Analysis (MCDA) techniques in aircraft design and aircraft evaluation decision making processes is one strategy to deal with multiple, conflicting criteria. The goal of this research is to investigate the approaches how existing MCDA techniques can be improved to better solve decision making problems, and how to implement the improved MCDA techniques in aircraft design and evaluation processes.

There are several MCDA techniques available to solve decision making problems, where different methods have different underlying assumptions, information requirements, and decision rules that are designed for solving a certain class of decision making problems. Thus, it is important to select the most appropriate MCDA method for a given problem. An advanced approach to effectively select the most appropriate MCDA method for a given problem is presented and an intelligent multi-criteria decision support system is developed.

The inherent uncertainties in the decision analysis process have crucial impacts on the final solution for a decision making problem. A new approach for uncertainty assessment is proposed. This approach consists of four steps: uncertainty characterization by percentage uncertainty with confidence level, uncertainty analysis using error propagation techniques, local

sensitivity analysis based on iterative binary search algorithm, and global sensitivity analysis using partial rank correlation coefficients. The proposed approach is implemented and an uncertainty assessment module is integrated into the developed intelligent multi-criteria decision support system.

The first proof of concept is the implementation of an improved MCDA method with uncertainty assessment in aircraft conceptual design process. A new optimization framework incorporating MCDA techniques in aircraft design process is established. The developed intelligent multi-criteria decision support system is used to select an appropriate MCDA method. It is demonstrated that the chosen MCDA method with improvement provides a better objective function for the optimization than the traditional weighted sum method. Furthermore, considering that the inherent uncertainties and subjectivities of the weighting factors have crucial impacts on the design solution, surrogate models for the multiple design criteria in terms of the weighting factors are constructed. Results show that the constructed surrogate models can enable efficient uncertainty assessment for the weighting factors.

The second proof of concept is the application of an appropriate MCDA method with uncertainty assessment in business aircraft evaluation process. The selection of the most appropriate MCDA method is conducted through the developed intelligent multi-criteria decision support system. In addition to the technical *hard* criteria, the soft criteria are considered to be the decisive factors in decision analysis process. In the business aircraft evaluation process, three soft criteria: passenger comfort level, product support level, and manufacturer's reputation, are considered and quantified. The synergy of technical *hard* criteria and additional soft criteria is the unique advantage of the MCDA techniques.

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Glossary

- ACJ: Airbus Corporate Jet
- AHP: Analytical Hierarchy Process
- AI: Appropriateness Index
- ANP: Analytical Network Process
- ATM: Air Traffic Management
- BBJ: Boeing Business Jet
- BCA: Business & Commercial Aviation
- CI: Consistency Index
- CL: Confidence Level
- CR: Consistency Ratio
- DLR: German Aerospace Center
- DM: Decision Maker
- DOC: Direct Operating Costs
- ELECTRE: Elimination and Choice Translation Reality
- EPNdB: Decibels of Effective Perceived Noise
- GA: Genetic Algorithms
- GUI: Graphical User Interface

- IFR: Instrument Flight Rules
- ITOPSIS: Improved TOPSIS
- LCA: Life Cycle Assessment
- LHS: Latin Hypercube Sampling
- MADM: Multi-Attribute Decision Making
- MCDA: Multi-Criteria Decision Analysis/Aid
- MCDM: Multi-Criteria Decision Making
- MODM: Multi-Objective Decision Making
- N/F: Non-Feasible
- NBAA: National Business Aviation Association
- OEM: Operating Empty Mass
- OR: Operational Research
- PN/F: Physically Non-Feasible
- PROMETHEE: Preference Ranking Organization METHod for Enrichment Evaluations
- RI: Random Consistency Index
- RMSE: Root Mean Square Error
- SAW: Simple Additive Weighting
- SMART: Simple Multi-Attribute Rating Technique
- SNR: Signal-to-Noise Ratio
- TOM: Take-off Mass
- TOPSIS: Technique for Order Preference by Similarity to Ideal Solution
- VAMPzero: Virtual Aircraft Multidisciplinary Analysis and Design Processes

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Introduction

The demands on air travel are increasing, not only regarding lower costs, but also better service quality, higher safety, and more environmental friendliness. The imperatives of air transport have evolved from *Higher, Further, Faster* to *More Affordable, Safer, Cleaner and Quieter* (1). In order to sustain the growth of air transport in a long term, the aerospace industry is faced with the challenge of designing more competitive aircraft satisfying these multiple criteria simultaneously.

As an important field in Operational Research (OR), Multi-Criteria Decision Analysis (MCDA) is a process that allows one to make decisions in the presence of multiple, potentially conflicting criteria (87). Common elements in the decision analysis process are a set of design alternatives, multiple decision criteria, and weighting factors reflecting the preference information of Decision Maker (DM). The MCDA techniques can help the DM to evaluate the overall performance of the design alternatives. Further, the MCDA techniques can provide aiding in the generation, analysis, and optimization of design solutions.

1.1 Motivation

The competitiveness of an aircraft is no longer dominated by economic criteria, such as purchase price and operating costs (26). Moreover, it is alerted that by applying classic Direct Operating Costs (DOC) comparisons as the only yardstick in the evaluation of an aircraft, manufacturers run the risk of designing aircraft types and capabilities not fully suited to satisfy long term transportation needs (58).

In addition to the economic consideration, there are several other criteria needed to be taken into account in aircraft design and evaluation decision making processes. For instance, environmental aspects and level of comfort. Continuous growth in passenger traffic and increasing public awareness of aircraft noise and emissions have made environmental considerations extremely critical in the design of future aircraft (5). Besides, passengers are more concerned about crowded flight and airlines are criticized for the increasing of load factors in order to fully utilized the capacity (76). Therefore, considering these multiple criteria simultaneously, aircraft design and aircraft evaluation are typical multi-criteria decision making problems and need to be prudently conducted.

Applying the MCDA techniques in aircraft design and aircraft evaluation processes is one strategy to deal with multiple, conflicting criteria. The MCDA techniques are utilized to aggregate the multiple design criteria into one composite figure of merit, which serves as an objective function in the optimization process. The MCDA techniques allow transparent trade-offs among criteria and support the designer to quickly assess the compromised design alternatives. Moreover, the MCDA techniques have the ability to handle large number of criteria in the design and evaluation processes.

Theory of the MCDA Techniques

Although MCDA as a discipline has a relatively short history of about 40 years, over 70 MCDA techniques have been developed for facilitating the decision making process (87). Among these 70 MCDA techniques, different methods have different underlying assumptions, information requirements, analysis models, and decision rules that are designed for solving a certain class of decision making problems. This implies that it is critical to select the most appropriate method to solve a given problem.

Decision criteria and weighting factors are main input data in the decision making process. It is observed that there are always uncertainties existing in the decision criteria due to incomplete information or limited knowledge, while the weighting factors are often highly subjective, considering the fact that they are elicited based on the DM's experience or intuition (7), (28). Therefore, uncertainty assessment for the decision criteria and the weighting factors should be prudently performed.

Practice of the MCDA Techniques in Aerospace Industry

Aerospace Systems Design Laboratory at Georgia Institute of Technology pioneered the application of the MCDA techniques in aerospace systems design. A probabilistic MCDA method for multi-objective optimization and product selection was developed (6). However, it was pointed that this method did not consider the absolute location of the joint probability distribution and the weighting factors were used to adjust the target values (49). Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was utilized to the selection of technology alternatives in conceptual and preliminary aircraft design (44). However, TOPSIS has the limitations that it assumes that each criterion's utility is monotonic and is rather sensitive to the weighting factors. A multi-criteria interactive decision-making advisor for the selection of the most appropriate decision making method was developed (48). However, only limited methods were implemented and the uncertainties propagated in the decision analysis process were not addressed explicitly.

Only limited research has been conducted to aircraft evaluation using the MCDA techniques. Four civil aircraft in terms of six criteria was evaluated by Simple Additive Weighting (SAW) (18). However, SAW is very sensitive to the normalization method and the weighting factors. Besides, civil aircraft was assessed by three criteria: DOC, operational commonality, and added values (58), (26). The added values were quantified by equivalent DOC based on the weighting factors. However, inherent subjectivity and uncertainty of the weighting factors detriments the usefulness of this approach. Further, seven initial training aircraft were evaluated by sixteen criteria using TOPSIS (82). However, only technical performance are considered because of the difficulty of collecting qualitative data. In addition, three MCDA methods: SAW, TOPSIS, and Analytic Hierarchy Process (AHP), were applied to an airport selection problem, where seven alternatives were evaluated by twelve criteria (40). The authors concluded that these three methods generated consistent result with the same weighting factors and suggested that the weighting factors should be considered more carefully.

In summary, although large efforts have been made to the application of the MCDA techniques, a large gap still exists between theory and practice, especially in the aerospace industry.

1.2 Research Statement

The goal of this research is to fill the gap by investigating how existing MCDA techniques can be improved to better solve decision making problems, and how to implement the improved MCDA techniques in aircraft design and evaluation processes. The following research objectives are considered critical to achieve the overall research goal:

1. Apply the most appropriate MCDA method for the decision making problem under consideration.

2. Assess the uncertainties propagated in the decision analysis process when applying the MCDA techniques.

3. Demonstrate the capabilities of the MCDA techniques with uncertainty assessment in aircraft design and aircraft evaluation processes.

The research objectives of this study can be best introduced through a series of research questions as follows:

- **Question 1:** How to select the most appropriate MCDA method for the decision making problem under consideration?

- **Question 2:** How to capture and assess the uncertainties propagated in the decision analysis process when solving decision making problems?

- **Question 3:** How to implement the MCDA techniques in aircraft design and aircraft evaluation processes?

In order to answer the research questions, several hypotheses are proposed:

- **Hypothesis 1:** It is feasible to quantify the appropriateness of the MCDA methods for a given decision making problem. (Question 1)

- **Hypothesis 2:** Statistical techniques are capable of effectively dealing with the uncertainties propagated in the decision analysis process. (Question 2)

- **Hypothesis 3:** It is beneficial to implement the MCDA techniques in aircraft design and aircraft evaluation processes. (Question 3)

1.3 Thesis Outline

The outline of the thesis is illustrated in Figure 1.1. In Chapter 2, an overview of the MCDA techniques is provided. An advanced approach to facilitate the selection of the most appropriate MCDA method is presented and an intelligent multi-criteria decision support system is developed in Chapter 3. Chapter 4 introduces a new uncertainty assessment approach in the decision analysis process. In Chapter 5, the implementation of an improved MCDA technique with uncertainty assessment in aircraft design is presented as the first proof of concept. In Chapter 6, business aircraft evaluation using an appropriate MCDA technique with uncertainty assessment is presented as the second proof of concept. The thesis is summarized and some recommendations for future work are given in Chapter 7.

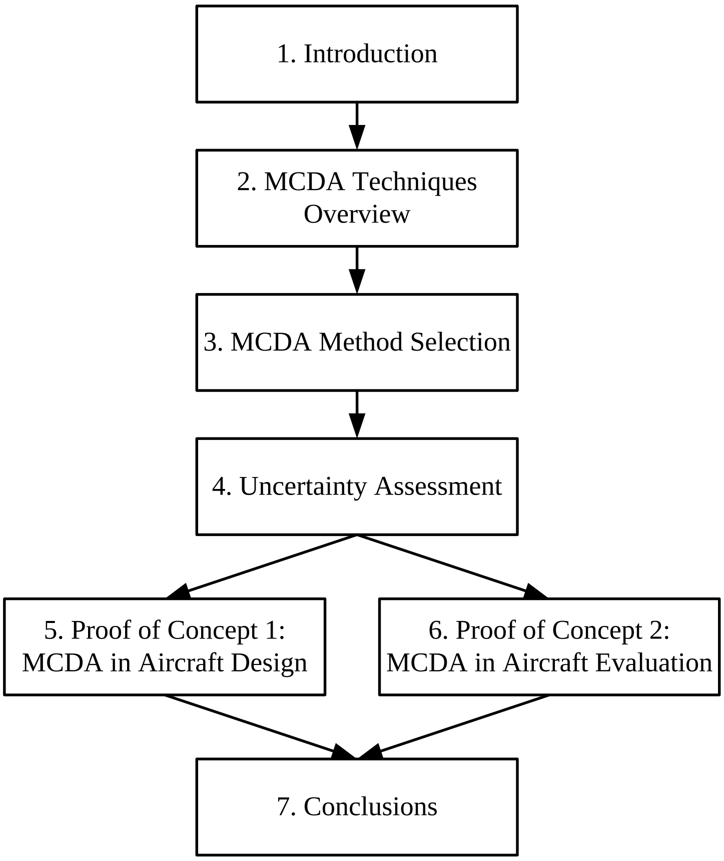

Figure 1.1: Thesis Outline

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Multi-Criteria Decision Analysis Techniques Overview

There are essentially two approaches to solve decision making problems: non-compensatory and compensatory methods (39). Non-compensatory methods do not permit trade-offs among criteria, while compensatory methods permit trade-offs among criteria. According to this classification, several widely used decision analysis methods are summarized in Table 2.1 and will be explained in detail in the following sections.

Table 2.1: Typical Non-compensatory and Compensatory Decision Analysis Methods (39)

Non-compensatory Methods	Compensatory Methods
Conjunctive method	Analytic hierarchy process
Disjunctive method	Expected utility theory
Dominance method	Multi-attribute utility theory
ELECTRE	Multiplicative weighting method
Elimination by aspects	PROMETHEE
Lexicographic method	Simple additive weighting
Maximin method	TOPSIS
Maximax method	

It is noted that ELECTRE is classified as one non-compensatory method (14), considering that the role of criteria weights in ELECTRE are *coefficients of importance* (68), (20). Besides, a poor criterion is judged irrespective to other good criteria, which distinguishes ELECTRE from compensatory methods (60).

2.1 Concepts and Terminologies

In order to have a universal understanding of the MCDA techniques, several important concepts and terminologies are introduced in this section.

MCDM and MCDA

There are two schools of decision analysis methods: Multi-Criteria Decision Making (MCDM) developed by the American school (85), and Multi-Criteria Decision Analysis/Aid (MCDA) created by the European school (67). Most researchers use MCDM and MCDA interchangeably (7), (87), (28). In this research, the European school (MCDA) is followed.

Criteria, Attributes, and Objectives

The distinctions among *criteria*, *attributes*, and *objectives* are made as follows (39).

- **Criteria**: A criterion is a measure of performance when evaluating an alternative.
- **Attributes**: An attribute is an inherent characteristic of an alternative.
- **Objectives**: An objective is something to be pursued to its fullest. It indicates the direction of change desired.

The relationship among *criteria*, *attributes*, and *objectives* are shown in Figure 2.1.

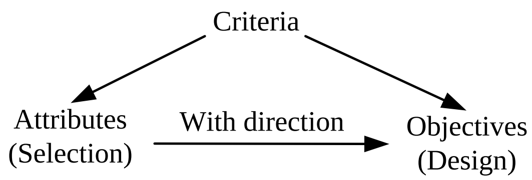


Figure 2.1: The Relationship Among Criteria, Attributes, and Objectives (73)

As shown in Figure 2.1, criteria are emerging as a form of attributes or objectives, and attributes with directions are objectives. For example, level of comfort is a criterion when evaluating an aircraft, cabin volume and noise are attributes of the aircraft which can be used to measure the level of comfort, the maximization of cabin volume and the minimization of noise are objectives in the aircraft design process.

Decision Matrix

At the heart of the MCDA techniques is the concept of decision matrix. Let A_i be the i-th alternative ($i = 1, 2, ..., m$) and x_j be the j-th criterion ($j = 1, 2, ..., n$). Suppose x_{ij} stands for the value of criterion x_j with respect to alternative A_i. Then, a quantitative MCDA problem of ranking or sorting m alternatives based on n criteria can be represented using decision matrix, as shown in Table 2.2.

Table 2.2: Decision Matrix

Alternatives	Criteria			
A_1	x_{11}	x_{12}	$\dots$	x_{1n}
A_2	x_{21}	x_{22}	$\dots$	x_{2n}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	x_{m1}	x_{m2}	$\dots$	x_{mn}

Preference Information

The preference information describes the DM's attitude in favor of one criterion over another when choosing between alternatives, usually in the form of weighting factors. Typical preference information elicitation techniques can be found in Appendix A.

Pareto Frontier

Pareto frontier is introduced to find the best compromised solution which has the maximum overall performance satisfying all the criteria simultaneously (38). In the feasible solution space, a solution is dominated if there is another solution which excels it in one or more criteria and equals it in the remainder (17). A non-dominated solution is one which no criteria can be improved without a simultaneous detriment to at least one of the others. A two-dimensional Pareto frontier for the minimization of two criteria is illustrated in Figure 2.2. It can be seen that Pareto frontier is composed of non-dominated solutions.

With these important concepts and terminologies, several typical decision analysis methods will be introduced in the following sections.

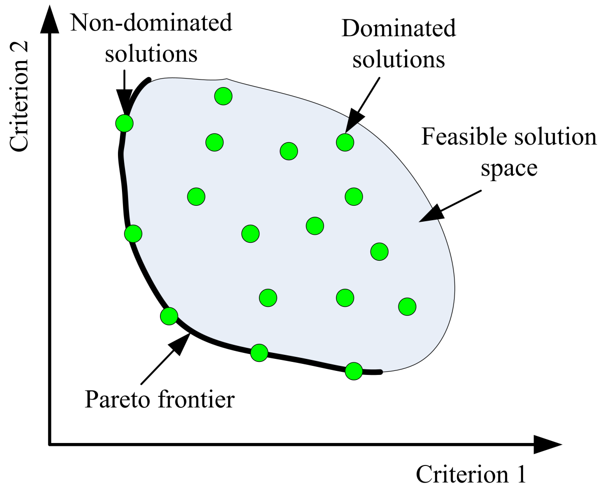


Figure 2.2: Pareto Frontier in Two Dimensions

2.2 Typical Non-compensatory Decision Analysis Methods

Non-compensatory decision analysis methods do not permit trade-offs between criteria, that is, a disadvantage in one criterion cannot be offset by an advantage in other criterion. The non-compensatory methods are credited for their simplicity. As summarized in Table 2.1, typical non-compensatory decision analysis methods are explained in detail in the following subsections.

2.2.1 Conjunctive Method

The DM sets up the acceptable minimal criteria values. Any alternative which has a criterion value less than the standard level will be rejected (39). The i-th alternative A_i $(i = 1, 2, ..., m)$ is classified as an acceptable alternative only if

$$x_{ij} \geq x_j^0, \quad j = 1, 2, ..., n \tag{2.1}$$

where x_j^0 is the standard level of the j-th criterion x_j, and bigger criteria values are preferred. The cutoff values given by the DM play a key role in eliminating the alternatives; if too high, none is left; if relatively low, several alternatives are left after filtering. Hence increasing the minimal standard levels in an iterative way, the alternatives can be narrowed down to a single choice.

The Conjunctive method does not require the criteria to be in numerical form, and the relative importance of the criteria is not needed. The Conjunctive method is not usually used for selection of alternatives but rather for dichotomizing them into acceptable and not acceptable categories.

2.2.2 Disjunctive Method

In the Disjunctive method, an alternative is evaluated on its greatest value of a criterion (39). The i-th alternative A_i $(i = 1, 2, ..., m)$ is classified as an acceptable alternative only if

$$x_{ij} \geq x_j^0, \quad j = 1 \text{ or } 2 \text{ or ... or } n \tag{2.2}$$

where x_j^0 is the desirable level of the j-th criterion x_j, and bigger criteria values are preferred.

As with the Conjunctive method, the Disjunctive method does not require the criteria to be in numerical form, and it does not need information on the relative importance of the criteria.

2.2.3 Dominance Method

In order to obtain a set of non-dominated solutions before the final choice, the Dominance method can be used to screen the alternatives. The Dominance method takes the following procedures (17) :

- Compare the first two alternatives and if one is dominated by the other, discard the dominated one.
- Next, compare the un-discarded alternative with the third alternative and discard any dominated alternative.
- Then, compare the fourth alternative and so on.
- After all the alternatives are compared, the non-dominated set is determined.

The Dominance method does not require any assumption or any transformation of criteria. The non-dominated set usually has multiple alternatives, hence, the Dominance method is mainly used for initial filtering.

2.2.4 ELECTRE

ELECTRE (Elimination and Choice Translation Reality) methods use the concept of outranking relation introduced by Benayoun (8). For instance, suppose there are m alternatives based on n evaluation criteria, with weighting factors $[w_1, w_2, ..., w_n]$, x_{ij} stands for the value of criterion x_j with respect to alternative A_i. An outranking relation between alternative A_k and alternative A_l $(k, l = 1, 2, ..., m, k \neq l)$ is defined as: A_k is preferred to A_l when A_k is at least as good as A_l with respect to a majority of criteria and when A_k is not significantly poor regarding any other criteria. After the assessment of the outranking relations for each pair of alternatives, dominated alternatives can be eliminated and non-dominated alternatives can be obtained for further consideration.

There are several different versions of ELECTRE methods, including ELECTRE I, IS, II, III, IV and TRI (68), (21). ELECTRE I is the first decision analysis method using the concept of outranking relation, the other versions of ELECTRE methods are extensions of ELECTRE I. In this subsection, the stepwise calculations of ELECTRE I will be described in detail and the other ELECTRE methods will be briefly introduced. ELECTRE I is composed of the following nine steps (39).

1. Normalize the decision matrix

$$R = \begin{bmatrix} r_{11} & r_{12} & ... & r_{1n} \\ r_{21} & r_{22} & ... & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & ... & r_{mn} \end{bmatrix}, r_{ij} = \frac{x_{ij}}{\sqrt{\sum\limits_{i=1}^{m} x_{ij}^2}}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.3}$$

2. Calculate the weighted normalized decision matrix.

$$V = RW = \begin{bmatrix} r_{11} & r_{12} & ... & r_{1n} \\ r_{21} & r_{22} & ... & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & ... & r_{mn} \end{bmatrix} \begin{bmatrix} w_1 & & & \\ & w_2 & & \\ & & \ddots & \\ & & & w_n \end{bmatrix} \tag{2.4}$$

3. Determine the concordance and discordance sets.

For each pair of alternatives A_k and A_l, the set of decision criteria $J = (j \mid j = 1, 2, ..., n)$ is divided into two disjoint subsets. The concordance set C_{kl} of A_k and A_l is composed of all criteria which support that A_k is preferred to A_l. The discordance set D_{kl} is the complementary subset of the concordance set C_{kl}. In other words, $D_{kl} = J - C_{kl}$.

$$\begin{array}{l} C_{kl} = \{ j \left| x_{kj} \geq x_{lj} \right. \}, \ (k, l = 1, 2, ..., m, \text{and} k \neq l) \\ D_{kl} = \{ j \left| x_{kj} < x_{lj} \right. \} = J - C_{kl} \end{array} \tag{2.5}$$

4. Calculate the concordance matrix C.

   The concordance index is calculated by the sum of the criteria weights which are contained in the concordance set. For example, the concordance index c_{kl} between A_k and A_l is calculated by Equation 2.7.

$$C = \begin{bmatrix} - & c_{12} & ... & c_{1n} \\ c_{21} & - & c_{23} & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & ... & - \end{bmatrix} \tag{2.6}$$

$$c_{kl} = \frac{\sum\limits_{j \in C_{kl}} w_j}{\sum\limits_{j=1}^{n} w_j} \tag{2.7}$$

5. Calculate the discordance matrix D.

   The discordance index reflects the degree to which one alternative is worse than the other. For instance, the discordance index d_{kl} between A_k and A_l is calculated by Equation 2.9.

$$D = \begin{bmatrix} - & d_{12} & ... & d_{1n} \\ d_{21} & - & d_{23} & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & ... & - \end{bmatrix} \tag{2.8}$$

$$d_{kl} = \frac{\max\limits_{j \in D_{kl}} |v_{kj} - v_{ij}|}{\max\limits_{j \in J} |v_{kj} - v_{ij}|} \tag{2.9}$$

It should be noticed that differences among criteria weights are contained in the concordance matrix C, while differences among criteria values are reflected in the discordance matrix D.

6. Determine the concordance dominance matrix.

   A concordance threshold c needs to be chosen to perform the concordance test. Alternative A_k possibly dominates alternative A_l, if the concordance index c_{kl} exceeds at least a certain threshold c, that is, $c_{kl} \geq c$.

   In ELECTRE I, a Boolean matrix is used to convert the concordance test into numerical values (0 and 1). If the concordance test is passed ($c_{kl} \geq c$), then the concordance index is 1. Otherwise, if the concordance test is failed ($c_{kl} < c$), the concordance index is 0.

7. Determine the discordance dominance matrix.

   A discordance threshold d needs to be chosen to perform the discordance test. Alternative A_k possibly dominates alternative A_l, if the discordance index d_{kl} is smaller than a certain threshold d, that is, $d_{kl} \leq d$.

   As with the case of the determination of the concordance dominance matrix, the discordance test is converted into numerical values (0 and 1) by a Boolean matrix. The discordance index is 1 when the discordance test is passed ($d_{kl} \leq d$), and it is 0 when the discordance test is failed ($d_{kl} > d$).

8. Aggregate the dominance matrix.

   An outranking relation can be justified only if both the concordance index and the discordance index do not violate their corresponding thresholds. That is, $c_{kl} \geq c$ and $d_{kl} \leq d$. The aggregated dominance matrix is calculated by an element-to-element product of the concordance dominance matrix and the discordance dominance matrix.

9. Eliminate the dominated alternatives.

   The aggregated dominance matrix gives the partial preference of the alternatives. In the aggregated dominance matrix, the element 1 in the column indicates that this alternative is dominated by other alternatives. Thus, any alternative which has at least one element of 1 in the column can be eliminated.

ELECTRE I is widely used because of its simple logic and refined computational procedures. However, the two concordance and discordance threshold values have significant impact on the final results. Additionally, the calculation procedures will become more complex as the increase of the dimension of decision matrix.

One Example of a Car Selection Problem using ELECTRE I

One example of a car selection problem using ELECTRE I is demonstrated in this subsection. Suppose that one DM wants to select a car with the consideration of three criteria: handling, fuel-economy, and power. Fuel-economy is one cost criterion (smaller value of fuel-economy is preferred), while handling and power are benefit criteria (bigger values of handling and power are preferred). There are three alternatives available: Ford, Lexus, and Saab. A ten-point score is assigned to the three criteria for each alternative, respectively. The weighting factors among the three criteria are [0.3 0.4 0.3]. The decision matrix is summarized in Table 4.6.

Table 2.3: The Decision Matrix of a Car Selection Problem using ELECTRE I

	Criteria		
	C_1: Handling	C_2: Fuel-economy	C_3: Power
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
A_1: Ford	8	7	10
A_2: Lexus	9	6	5
A_3: Saab	6	7	8

Given the decision matrix shown in Table 4.6, going through the described nine-step calculations of ELECTRE I, the aggregated dominance matrix is shown in matrix M.

$$M = \begin{array}{cc} & \text{is dominated by} \\ \text{dominates} & \begin{bmatrix} - & 0 & 1 \\ 0 & - & 0 \\ 0 & 0 & - \end{bmatrix} \end{array}$$

In the aggregated dominance matrix M, the element 1 in the column indicates that this alternative is dominated by other alternatives. Thus, A_3 is dominated by A_1 and A_2. In another words, A_1 and A_2 are non-dominated alternatives. Therefore, in this car selection problem using ELECTRE I, A_3 (Saab) should be eliminated from the candidate cars, A_1 (Ford) and A_2 (Lexus) can be recommended for further consideration.

ELECTRE IS

ELECTRE IS is similar to ELECTRE I, except that in Step 6 (Determine the concordance dominance matrix), interval values between 0 and 1 are used instead of Boolean numbers (0 or 1) (68), (21), (60). In order to discriminate between two alternatives, two thresholds have to be defined for each criterion: indifference threshold and strict preference threshold.

ELECTRE II

ELECTRE II is also similar to ELECTRE I. The main difference lies in the definition of two outranking relations: strong outranking and weak outranking. For each criterion, two strong outranking thresholds and one weak outranking threshold have to be defined.

ELECTRE III

ELECTRE III uses the same principle of ELECTRE II. For each criterion, an indifference threshold, a preference threshold, and a veto threshold have to be defined in order to compare the alternatives. Both the concordance dominance matrix and discordance dominance matrix are constructed by interval values between 0 and 1. The aggregation of the concordance dominance matrix and discordance dominance matrix is obtained by a credibility matrix. The final classification of alternatives is based on ascending and descending distillations (68), (21).

ELECTRE IV

Unlike the previously described ELECTRE methods, ELECTRE IV does not require criteria weights in the calculation procedures. Instead, it uses the number of criteria in different preference areas. For each criterion, an indifference threshold, a preference threshold, and a veto threshold are required in order to compare the alternatives. Similar to ELECTRE III, a credibility matrix is calculated, and the classification of alternatives is based on ascending and descending distillations.

ELECTRE TRI

In ELECTRE TRI, some reference alternatives are introduced, all alternatives are compared to these reference alternatives. Similar to ELECTRE III, a credibility matrix is computed with respect to reference alternatives. The outranking relations between candidate alternatives and reference alternatives are established using the credibility matrix and a veto threshold. ELECTRE TRI can reduce the computational cost of alternative comparisons when the number of alternatives is large.

Summary of ELECTRE Methods

The main characteristics of all versions of ELECTRE methods were summarized by Roy (68), as shown in Table 2.4. Considering different problem statements, some guidelines on how to choose among ELECTRE methods were also suggested. For instance, if it is truly essential to work with a very simple method and it is realistic to have no information on the indifference threshold and preference threshold, ELECTRE I should be selected in order to eliminate the non-dominated alternatives, while ELECTRE II should be used in order to build a partial pre-order of alternatives. ELECTRE VI would be convenient only if there exists a good reason to refusing the introduction of importance coefficients. In general, ELECTRE IS, II, III, IV, and TRI do provide powerful support for the classification of the alternatives. However, they require too many threshold definitions from DMs, thus, it is rather complex to implement these methods in real world problems (60).

Table 2.4: Main Characteristics of ELECTRE Methods (68)

ELECTRE methods	I	IS	II	III	IV	TRI
Require indifference and preference thresholds	no	yes	no	yes	yes	yes
Require criteria weights	yes	yes	yes	yes	no	yes
Outranking relations	binary	binary	strong and weak	interval values	strictly, weakly, hardly preferred, or indifferent	interval values

2.2.5 Elimination By Aspects Method

In this method, the DM is assumed to have minimum cutoffs for each criterion. A criterion is selected, and all alternatives which do not pass the cutoff on that criterion are eliminated. Then another criterion is selected, and so forth. The process continues until all alternatives but one are eliminated (39).

The elimination by aspects method eliminates alternatives which do not satisfy some standard level, and it continues until all alternatives except one have been eliminated. However, only small part of the information is used when comparing the alternatives.

2.2.6 Lexicographic Method

In the Lexicographic method, the DM compares the alternatives on the most important criterion. If one alternative has a better criterion value than any of the other alternatives, the alternative is chosen and the decision process ends. However, if some alternatives are tied on the most important criterion, the subset of tied alternatives is then compared on the next most important criterion. The process continues sequentially until a single alternative is chosen or until all the criteria have been considered.

The Lexicographic method does not require comparability across criteria, and the preference information on the criteria is not necessarily in numerical values. However, it only utilizes a small part of the available information in making a final decision.

2.2.7 Maximin Method

In the Maximin method, the overall performance of an alternative is determined by the weakest or poorest criterion. The DM examines the criteria values for each alternative, note the worst value for each alternative, and then select the alternative with the most acceptable value in its worst criterion. It is the selection of the maximum (across alternatives) of minimum (across criteria) values, or the maximin (39). Mathematically speaking, the alternative A^* is selected such that

$$A^* = \left\{ A_i \left| \max_i \min_j r_{ij} \right. \right\}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.10}$$

where r_{ij} are normalized criteria values, and bigger criteria values are preferred.

2.2.8 Maximax Method

In contrast to the Maximin method, the Maximax method selects an alternative by its best criterion value rather than its worst criterion value. In this method, the best criterion value for each alternative is identified, then these maximum values are compared in order to select the alternative with the best value (39). Mathematically speaking, the alternative A^* is selected such that

$$A^* = \left\{ A_i \middle| \max_i \max_j r_{ij} \right\}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.11}$$

where r_{ij} are normalized criteria values, and bigger criteria values are preferred.

The Maximin method and the Maximax method are widely used in game theory. However, they utilize only a small part of the available information in making a final choice (only one criterion per alternative). The applicability of the Maximin method and the Maximax method is relatively limited.

2.3 Typical Compensatory Decision Analysis Methods

Compensatory decision analysis methods permit trade-offs between criteria, that is, small changes in one criterion can be offset by opposing changes in any other criteria. As summarized in Table 2.1, typical compensatory decision analysis methods are explained in detail in the following subsections.

2.3.1 Analytic Hierarchy Process

Analytic Hierarchy Process (AHP) was proposed to deal with decision making problems that have hierarchical structures of attributes (70). AHP is based on the idea of translating the hierarchy problem to a series of pairwise comparison matrices and obtaining the preference information for the attributes using eigenvector method. As one popular preference information elicitation techniques, the eigenvector method is explained in Appendix A.2. The first part of this subsection introduces the pairwise comparison matrix, followed by the computational steps of AHP.

Pairwise Comparison Matrix

The pairwise comparison concept originated from an experiment considering the subject of stimuli and responses performed by Weber in 1846. Weber stated that change in sensation was noticed when the stimulus was increased by a constant percentage of the stimulus itself. A nine-point scale based on Weber's law was created and shown in Table 2.5.

Table 2.5: Pairwise Comparison Scale (70)

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective.
3	Moderate importance of one over another	Experience and judgment slightly favor one activity over another.
5	Strong importance	Experience and judgment strongly favor one activity over another.
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice.
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation.
Reciprocals of above	If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i.	A reasonable assumption.

Suppose there are m alternatives and n criteria in a given problem. A pairwise comparison matrix is a m by m matrix, whose element y_{ij} indicates the DM's preference information of alternative i over alternative j for a given criterion. In total, there will be n $m \times m$ comparison matrices, as shown in Equation 2.12.

$$M = \begin{bmatrix} 1 & y_{12} & \cdots & y_{1m} \\ y_{21} & 1 & \cdots & y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m1} & y_{m2} & \cdots & 1 \end{bmatrix} \tag{2.12}$$

Computational Steps of AHP

1. Establish the decision making problem in a hierarchy structure.

2. Formulate the pairwise comparison matrix, as shown in Equation 2.12, for elements at a single level of the hierarchy with respect to each of the elements at a level immediately above.

3. Generate the weights of elements using the eigenvector method, as described in Appendix A.2. This procedure is repeated until all the weights of elements are obtained.

4. The alternative with a larger relative value is more favorable.

AHP provides a simple way to formulate a decision making problem and to elicit preference information, as it only requires pairwise comparisons between criteria or alternatives. However, it has some limitations. The preference independence among all elements at any level except for the bottom level is assumed. It would be problematic to use AHP where the criteria at the same level have correlated dependence. Another limitation in AHP is that the pairwise comparison matrix is required with each element describing the relative importance of an criterion over all other criteria or the relative preference of an alternative over all other alternatives. The complete pairwise comparison is not a trivial task for the DM and may trigger inconsistency problems, which will become worse with the increasing dimension of the pairwise comparison matrix.

2.3.2 Expected Utility Theory

Expected utility can be dated back to Daniel Bernoulli's resolution to the St. Petersburg paradox in 1738 (22), (25). The rule of the St. Petersburg game is that the player tosses a fair coin until *head* shows up for the first time, if this occurs at k-th toss, the payoff is 2^k guilders. The expected monetary value is $\sum_{i=1}^{n}(\frac{1}{2})^k 2^k = 1 + 1 + 1 + ... = \infty$. The people were asked how much they would pay for the game? However, the paradox is that no reasonable people would want to pay even small amount of money for the game with infinite expected value.

Bernoulli used a logarithmic utility index defined over wealth to compute a finite price for a gamble with an unbounded expected value, with the argumentation that the people estimate the game in terms of the utility of money outcomes, and the marginal utility is diminishing. For a person with present wealth a, the expected utility of the game is calculated by Equation 2.13 (25).

$$\sum_i p_i \log(a + x_i) \tag{2.13}$$

where p_i is the probability of the i-th game, and x_i is the outcome of the i-th game.

The value of the game with fixed amount v is calculated by $\log(a+v) = \sum_i p_i \log(a + x_i)$ and is shown in Equation 2.14 (25)

$$v = \prod_i (a + x_i)^{p_i} - a \tag{2.14}$$

Expected utility theory state that the DM chooses between risky prospects by comparing their expected utility values, which are calculated by the weighted sum of utility values of outcomes multiplied by their probabilities, as shown in Equation 2.15.

$$E(u|p, X) = \sum_{x \in X} p(x)u(x) \tag{2.15}$$

where x is a particular outcome from the set of all possible outcomes X, $p(x)$ is the probability of the particular come, $u(x)$ is its utility function.

Expected utility theory is suitable for decision making problems with risk and uncertainty. However, it is difficult to obtain an accurate utility function for each criterion, and the consistency of the utility functions among different criteria is hard to maintain.

2.3.3 Multi-Attribute Utility Theory

This method is based on the concept of utility function, which represents a mapping from the DM's preference into a mathematical function (43). The most widely used form is the additive multi-attribute utility method given by Equation 2.16, with two assumptions stating that the utility functions of all the attributes are independent and the relative weight of an attribute can be determined regardless of the relative weights of other attributes.

$$U\left(x_1, x_2, ..., x_n\right) = \sum_{i=1}^{n} w_i u_i\left(x_i\right) \tag{2.16}$$

where $[w_1, w_2, ..., w_n]^{\mathrm{T}}$ are weighting factors, $u_i(x_i)$ is the corresponding utility function of the i-th attribute x_i .

The additive multi-attribute utility provides utility function to represent the DM's preference information. However, the two assumptions including the independence of utility function and weights do not hold true for many practical decision making problems, which limits the use of this method.

2.3.4 Multiplicative Weighting Method

In this method, the relative weights $[w_1, w_2, ..., w_n]^{\mathrm{T}}$ are assigned to the criteria by the DM, the criterion values for each alternative are multiplied, with the relative weights as exponents. This method chooses the most preferred alternative which has the biggest value, as shown in Equation 2.17, when bigger criteria values are preferred.

$$A^* = \left\{ A_i \left| \max_i \prod_{j=1}^{n} x_{ij}^{w_j} \right. \right\}, i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.17}$$

Considering the exponentiation property, all criteria values should be greater than one in order to assure its monotonicity. When the criterion values are smaller than one, 10^k should be multiplied to all criterion values, where k is an exponent which make the smallest criterion value bigger than one.

2.3.5 PROMETHEE

In PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations) method (15), (16), a valued preference relationship based on a generalization of the notion of criteria is constructed first, and a preference index is defined and a valued outranking graph is obtained. According to the preference index, PROMETHEE I provides a partial preorder and PROMETHEE II offers a complete preorder on all actions (alternatives).

Criteria Generalization

The definition of the valued preference relationship between two actions a and b is described as follows (16):

- $P(a, b) = 0$ means an indifference between a and b.
- $P(a, b) \approx 0$ means weak preference of a over b.

- $P(a,b) \approx 1$ means strong preference of a over b.
- $P(a,b) = 1$ means strict preference of a over b.

For each criterion, a generalized criterion and a corresponding preference function are considered. In PROMETHEE, six types of generalized criteria are provided, as illustrated in Figure 2.3, where d is the difference between two criteria, p is the strict preference threshold, and q is the indifference threshold, s is the standard deviation in Gaussian distribution.

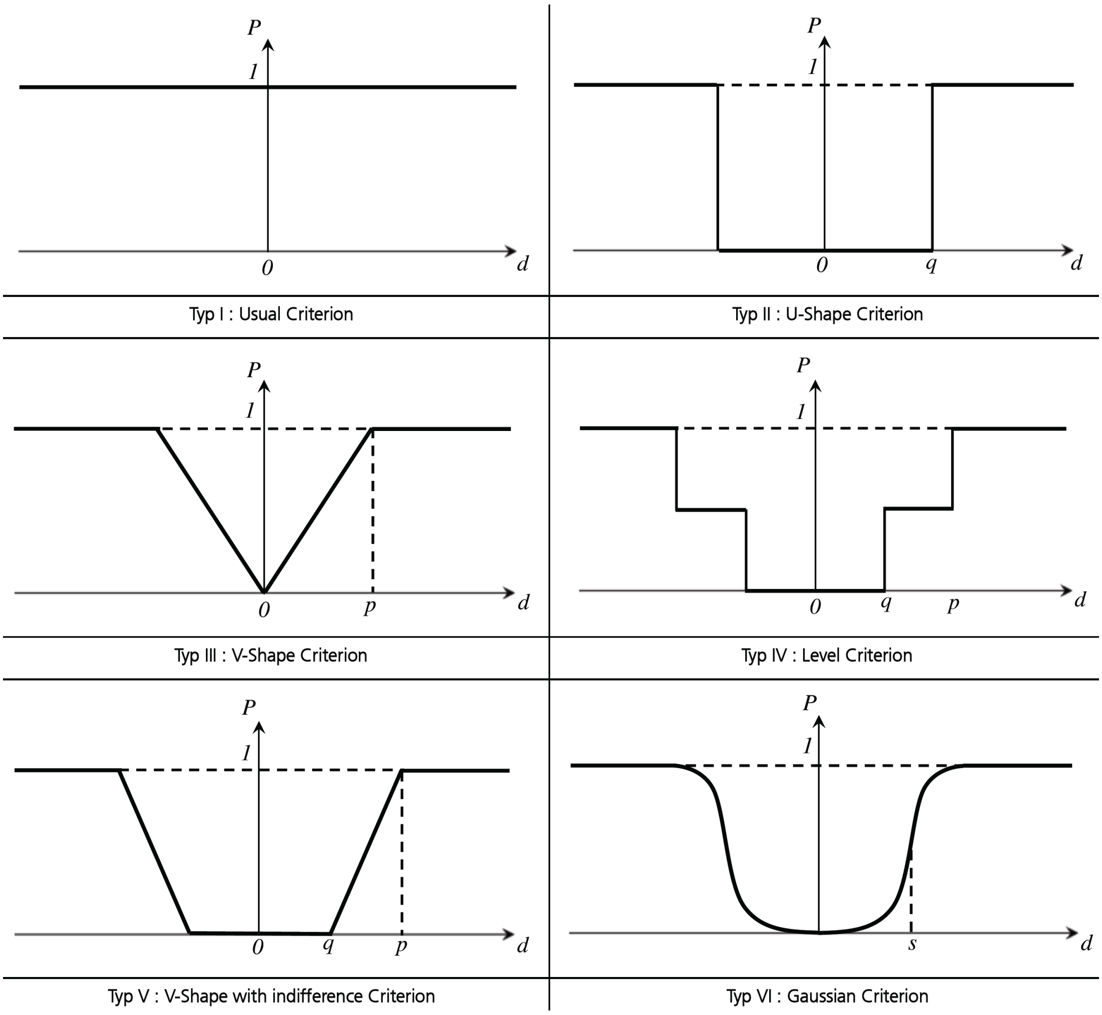


Figure 2.3: Six Types of Generalized Criteria (16)

Multi-Criteria Preference Index

The multi-criteria preference index of action a over action b, denoted by $\Pi(a, b)$, is defined as in Equation 2.18

$$\Pi(a, b) = \frac{\sum_{i=1}^{n} w_i P_i(a, b)}{\sum_{i=1}^{n} w_i} \tag{2.18}$$

where n is the number of criteria, w_i is the weighting factor of the i-th criterion, and P_i is the preference function of the i-th criterion. The multi-criteria preference index ranges from 0 to 1, with $\Pi(a, b) \approx 0$ represents a weak preference of action a over action b, and $\Pi(a, b) \approx 1$ represents a strong preference of action a over action b.

PROMETHEE Rankings

A positive outranking flow is defined by Equation 2.19 and a negative outranking flow is defined by Equation 2.20, respectively. Besides, a net outranking flow is calculated by Equation 2.21.

$$\Phi^+(a) = \sum_{b \in A} \Pi(a, b) \tag{2.19}$$

$$\Phi^-(a) = \sum_{b \in A} \Pi(b, a) \tag{2.20}$$

$$\Phi(a) = \Phi^+(a) - \Phi^-(a) \tag{2.21}$$

Based on Equation 2.19 and Equation 2.20, PROMETHEE I provides a partial preorder by considering the intersection of the positive outranking flow and negative outranking flow, which is listed as follows.

- Action a outranks action b, if $\Phi^+(a) \geq \Phi^+(b)$ and $\Phi^-(a) \leq \Phi^-(b)$.
- Action a is indifferent from action b, if $\Phi^+(a) = \Phi^+(b)$ and $\Phi^-(a) = \Phi^-(b)$.
- Otherwise, action a and action b are incomparable.

Based on Equation 2.21, PROMETHEE II considers action a outranks action b if $\Phi(a) > \Phi(b)$, and action a is indifferent from action b if $\Phi(a) = \Phi(b)$.

The six types of preference function and the partial or complete preorder in PROMETHEE provides the DM more insights in solving the given problem. However, in order to define the preference function, it requires too many threshold parameters. Moreover, these threshold parameters are rather subjective and different DMs often have different threshold values, which increases the complexity of the problem significantly.

2.3.6 Simple Additive Weighting

In Simple Additive Weighting (SAW) method (39), the relative weights $[w_1, w_2, ..., w_n]^{\mathrm{T}}$ are assigned to the criteria by the DM. The multiple criteria values together with their corresponding weights are aggregated into a single performance metric. SAW chooses the most preferred alternative A^* which has the maximum weighted average outcome, as shown in Equation 2.22, when bigger criteria values are preferred.

$$A^* = \left\{ A_i \,\middle|\, \max_i \sum_{j=1}^{n} w_j x_{ij} \right\}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.22}$$

SAW is one of the most widely known decision analysis methods because of its simplicity to understand and use. However, it also has some disadvantages. SAW requires all the criterion values to be both numerical and comparable, which will trigger the quantification problem of the qualitative criteria and normalization problem of all the elements in decision matrix. The quantification methods and normalization methods will have a significant influence on the final decision results. Moreover, SAW is sensitive to the weighting factors.

2.3.7 TOPSIS

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is based on the idea that the chosen alternative should have the shortest distance to the positive ideal solution and the furthest distance from the negative ideal solution. The distance is in the form of Euclidean distance (39), as shown in Figure 2.4.

TOPSIS requires decision matrix and relative weights as input data, its computational steps are summarized as follows.

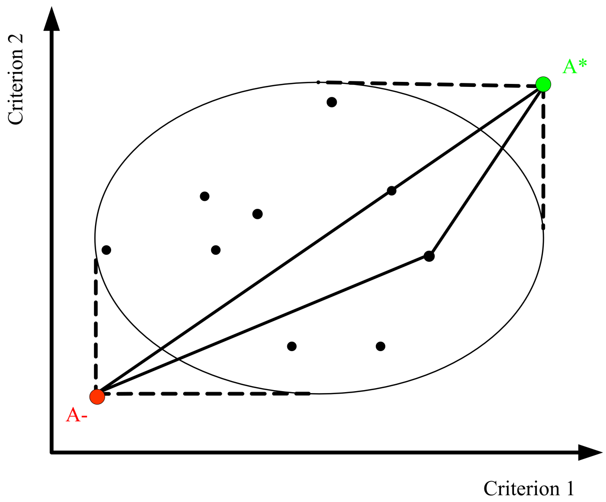


Figure 2.4: TOPSIS Method (39)

1. Normalize the decision matrix.

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.23}$$

2. Calculate the weighted normalized decision matrix.

$$r_{ij} = w_j z_{ij}, \quad i = 1, 2, ..., m, j = 1, 2, ..., n \tag{2.24}$$

3. Identify the positive ideal solution A^* and the negative ideal solution A^-.

$$A^* = \left\{ \left(\max_i r_{ij} | j \in J \right), \left(\min_i r_{ij} | j \in \hat{J} \right) | i = 1, 2, ..., m \right\} = \{x_1^*, x_2^*, ..., x_n^*\} \tag{2.25}$$

$$A^- = \left\{ \left(\min_i r_{ij} | j \in J \right), \left(\max_i r_{ij} | j \in \hat{J} \right) | i = 1, 2, ..., m \right\} = \{x_1^-, x_2^-, ..., x_n^-\} \tag{2.26}$$

where J is the benefit criteria set (bigger criterion value is preferred), and $\hat{J}$ is the cost criteria set (smaller criterion value is preferred). Thus, the positive ideal solution is composed of the maximum values of benefit criteria and the minimum values of cost criteria; while the negative ideal solution is composed of the minimum values of benefit criteria and the maximum values of cost criteria.

4. Calculate the distance of each alternative to the positive ideal solution and the negative ideal solution, respectively.

$$S_i^* = \sqrt{\sum_{j=1}^{k} \left(r_{ij} - x_j^* \right)^2}, \quad i = 1, 2, ..., m \tag{2.27}$$

$$S_i^- = \sqrt{\sum_{j=1}^{k} \left(r_{ij} - x_j^- \right)^2}, \quad i = 1, 2, ..., m \tag{2.28}$$

5. Calculate the relative closeness of each alternative to the ideal solutions.

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^*}, \quad i = 1, 2, ..., m \tag{2.29}$$

6. Rank the alternatives according to the value of C_i^* .

   TOPSIS suggests the best alternative which has the furthest distance from the negative ideal solution (biggest value of S_i^-) and shortest distance to the positive ideal solution (smallest value of S_i^*), thus, the increase of numerator and the decrease of denominator will lead to a bigger value of C_i^* in Equation 2.29. In other words, the alternative which maximizes the value of C_i^* ranks first.

Furthermore, in addition to Equation 2.29, the relative closeness of each alternative to the ideal solutions could be also aggregated by Equation 2.30.

$$C_i^- = \frac{S_i^*}{S_i^* + S_i^-}, \quad i = 1, 2, ..., m \tag{2.30}$$

where the decrease of numerator and the increase of denominator will result in a smaller value of C_i^-. Thus, the alternative which minimizes the value of C_i^- ranks first.

Besides, it is interesting to notice that the relationship between Equation 2.29 and Equation 2.30.

$$C_i^* + C_i^- = 1, \quad i = 1, 2, ..., m \tag{2.31}$$

Another approach is to visualize the relative closeness of each alternative to the ideal solutions via Pareto frontier, as illustrated in Figure 2.5, where the horizontal coordinate represents the distance to the positive ideal solution (S_i^*), while the vertical coordinate stands for the distance to the negative ideal solution with minus signal ($-S_i^-$). The minus signal is used to convert the preference direction of S_i^- for the convenience of displaying Pareto frontier.

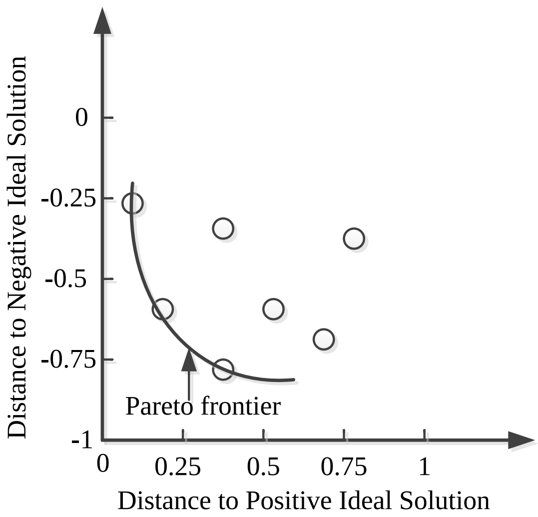


Figure 2.5: Pareto Frontier for the Relative Closeness to the Ideal Solutions in TOPSIS

The Pareto frontier approach does not need to aggregate the relative closeness to the ideal solutions, however, instead of one best alternative, a set of non-dominated alternatives are often obtained.

TOPSIS is one of the widely used compensatory decision analysis methods considering its simplicity and systematic calculation procedures. However, it has some limitations. TOPSIS assumes that each criterion's utility is monotonic, which is not appropriate for problems where a particular criterion value is desired to be achieved (39). TOPSIS is also rather sensitive to the weighting factors.

One Example of a Car Selection Problem using TOPSIS

In this subsection, TOPSIS is used to solve a car selection problem, as described in Subsection 2.2.4. The decision matrix shown in Table 4.6 is repeated here for the convenience of calculation.

Given the decision matrix summarized in Table 4.6, going through the described

Table 2.6: The Decision Matrix of a Car Selection Problem using TOPSIS

	Criteria		
	C_1: Handling	C_2: Fuel-economy	C_3: Power
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
A_1: Ford	8	7	10
A_2: Lexus	9	6	5
A_3: Saab	6	7	8

six-step calculations of TOPSIS, the relative closeness aggregated by Equation 2.29 is shown in C^*. Considering that the alternative which maximizes the value of C^* ranks first, thus, A_1 (Ford) is recommended as the best alternative for the DM.

$$C^* = \begin{bmatrix} 0.5175 \\ 0.4866 \\ 0.5043 \end{bmatrix}$$

Furthermore, the relative closeness aggregated by Equation 2.30 is shown in C^-. In this case, the alternative which has the smallest value of C^- ranks first. Therefore, A_1 (Ford) is ranked as the best alternative for the DM.

$$C^- = \begin{bmatrix} 0.4825 \\ 0.5134 \\ 0.4957 \end{bmatrix}$$

The Pareto frontier for the relative closeness to the ideal solutions is illustrated in Figure 2.6. It can be observed that A_1 (Ford) is the non-dominated alternative.

In summary, in this car selection example using TOPSIS, three approaches of representing the relative closeness of each alternative to the ideal solutions: aggregation by Equation 2.29 and Equation 2.30, and visualization via Pareto frontier, generate consistent result that A_1 (Ford) is the best alternative for the DM among the three candidate cars.

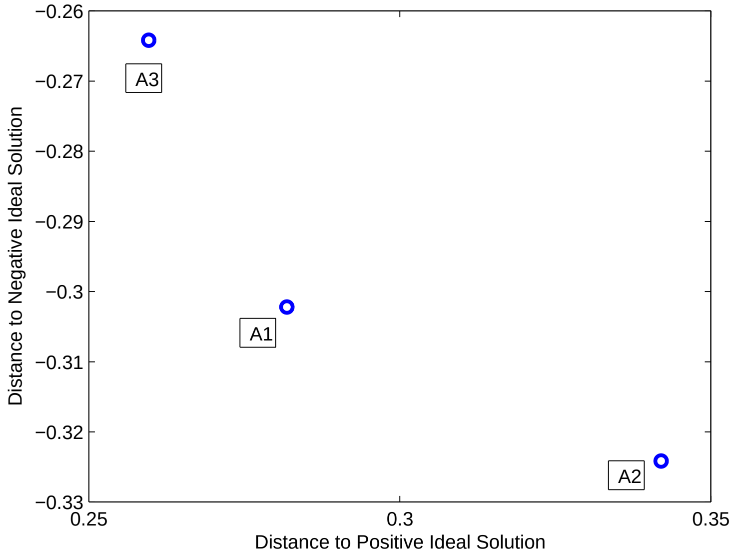


Figure 2.6: Pareto Frontier for the Relative Closeness to the Ideal Solutions in the Car Selection Example

3

MCDA Method Selection

The first objective of this research is the development of an intelligent multi-criteria decision support system in order to facilitate the selection of the most appropriate MCDA method for the problem under consideration effectively. In this chapter, with the perspective that the method selection itself is a complicated MCDA problem, twelve evaluation criteria are proposed to assess sixteen widely used MCDA methods. An Appropriateness Index (AI) is used to evaluate the methods and identify the most suitable one. This method selection approach is implemented and an intelligent multi-criteria decision support system is developed in MATLAB.

The framework of MCDA method selection was originally developed by (48). In this research, this framework has been successfully improved in order to yield more accurate and reliable solutions (77). Three major improvements are listed as follows.

1. The distinction between filter questions to screen out inappropriate methods in the initial step of selection, and scoring questions which are used as the attributes of a MCDA formulation and as the input data for method selection.

2. Methodology instructions for all sixteen widely used MCDA methods.

3. Most importantly, the newly developed uncertainty assessment module, which will be discussed in detail in Chapter 4.

3.1 Method Selection Background

Although MCDA has a relatively short history of about 40 years, over 70 MCDA techniques have been developed for facilitating the decision making process (73),(81),(87). Among these developed MCDA methods, different methods have different underlying assumptions, information requirements, analysis models, and decision rules that are designed for solving a certain class of decision making problems. It is critical to select the most appropriate method to solve the problem under consideration since the use of unsuitable methods might lead to misleading decisions. It can be seen that the selection of MCDA methods itself is a complicated MCDA problem (39) and needs to be prudently performed.

Over the past decades, considerable research has been conducted to deal with the selection of the most appropriate MCDA method for a given decision making problem. MacCrimmon firstly recognized the importance of MCDA method selection. He proposed a taxonomy of MCDA methods, created a method specification chart in the form of a tree diagram and provided an illustrative application example (53). Hwang developed another tree diagram, which consists of nodes and branches connected by choice rules that can be used for selecting the decision making method for a specified problem (39). Sen and Yang developed similar tree diagrams to help the DM with selecting the appropriate MCDA methods, and the selection was based on the type of preference information elicited (73). The tree diagram approach provides reasonable classification schemes and is easy to utilize. However, this approach has its own disadvantages: it usually gives two or more MCDA methods rather than the most appropriate method, and it only considers limited types of decision problems, preference information, and available methods. These limitations stop the tree diagram approach from being an effective solution to the method selection problem (66).

Possible criteria for evaluating MCDA methods were proposed as an alternative solution to the method selection problem. Tecle and Duckstein developed an approach based upon a composite programming algorithm which aided in selecting an appropriate MCDA method (79). They proposed four categories of criteria: DM-related characteristics, method-related characteristics, problem-related characteristics, and solution-related characteristics to evaluate the decision making methods. The independent criteria categories enable the DM to conduct the evaluation in a specified order. However, it is difficult to quantify all MCDA methods in terms of these four criteria categories. And by using these approaches, different users may get totally different results because the user's knowledge about the MCDA methods has a strong impact on the final results.

Artificial intelligence techniques were employed by Poh and Lu et. al. (64),(52) to help the DM select a MCDA method based on a series of user inputs. Poh suggested a knowledge-based system, which allowed the DM to select the most appropriate method among available 11 multi-attribute decision making methods. Lu et al. proposed an intelligent system, which facilitated selecting the most suitable method among seven multi-objective decision making methods. The knowledge-based intelligent system simplifies the methods selection problem with simple questions by allowing direct selection or automated selection based on DM's inputs. However, they do not clearly state the limitations or failure modes of the systems (66).

Although the tree diagrams approach, the criteria approach, and the artificial intelligent systems provide some capabilities to find the suitable decision making methods for a given problem, they have their own disadvantages. Therefore, an advanced approach with more capabilities needs to be developed to facilitate the MCDA method selection.

3.2 An Advanced Approach to Method Selection

To effectively select the most appropriate MCDA method for a given decision making problem, a systematic framework is proposed in this study. The proposed approach consists of eight steps: define the problem, define the evaluation criteria, perform initial screening, define the preferences on evaluation criteria, define the MCDA method for selection, evaluate the MCDA methods, choose the most suitable method, and conduct sensitivity analysis. This framework is illustrated in Figure 3.1. Each step of the proposed approach to method selection is discussed in detail in the following subsections.

3.2.1 Step 1: Define the Problem

The characteristics of the decision making problem under consideration are addressed in the problem definition step, such as identifying the number of alternatives, attributes, and constraints. The available information about the decision making problem is the

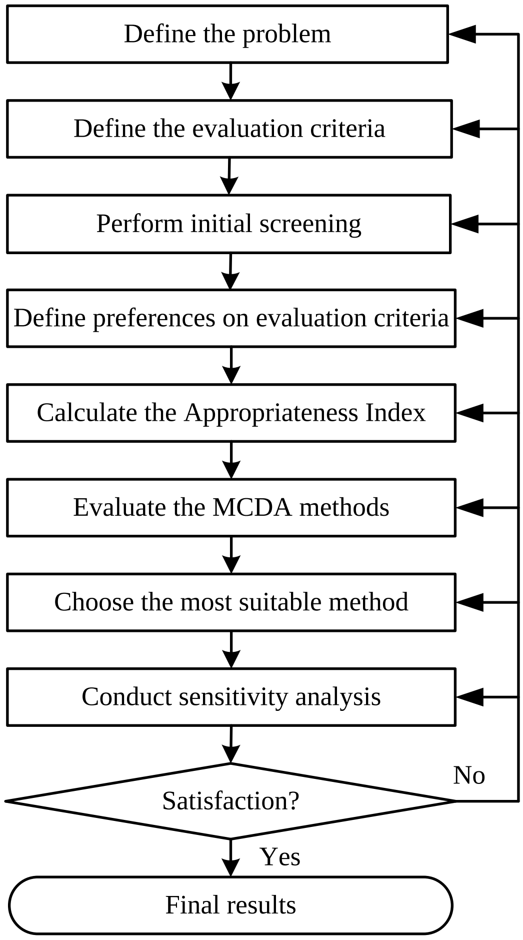


Figure 3.1: An Advanced Approach to MCDA Method Selection

basis on which the most appropriate MCDA techniques will be selected and utilized to solve the problem.

3.2.2 Step 2: Define the Evaluation Criteria

The proper determination of the applicable evaluation criteria is important because they have great influence on the outcome of the MCDA method selection process. However, simply using every criterion in the selection process is not the best approach because the more criteria used, the more information is required, which will result in higher computational cost. In this study, the characteristics of the MCDA methods are identified by the relevant evaluation criteria in the form of a questionnaire. Twelve questions are defined to capture the advantages, disadvantages, applicability, and computational complexity of each MCDA method.

- **Filter Questions**

  1. Is the method able to handle selection or optimization problems?
  2. Does the method allow trade-offs among criteria?
  3. What input data is required by the method?

- **Scoring Questions**

  4. What preference information does the method need?
  5. What decision rule does the method use to rank or sort the alternatives?
  6. Does the method evaluate the feasibility of the alternatives?
  7. Can the method handle any subjective attribute?
  8. Does the method handle qualitative or quantitative data?
  9. Does the method deal with discrete or continuous data?
  10. Can the method handle the problem with hierarchy structure of attributes?
  11. Is the method able to capture uncertainties existing in the problem?
  12. Can the method support visual analytics?

It should be noted that the first three filter questions will be used to screen out inappropriate methods in the initial step of selection, the other nine scoring questions will be used as the attributes of a MCDA formulation and as the input data of decision matrix for method selection.

3.2.3 Step 3: Perform Initial Screening

In the initial screening step, the first three filter questions are utilized to screen out inappropriate methods. For the first filter question, only the scoring MCDA methods are suitable for solving optimization problems since the scores aggregated by MCDA methods will serve as an objective function to be optimized in the problem solving process, while the classification MCDA methods are not suitable since they cannot offer an aggregated objective function for optimization.

For the second filter question, if trade-offs among criteria are allowable, all non-compensatory methods will be removed, and only compensatory methods remain as the candidate methods for further selection.

For the third filter question, most MCDA methods require decision matrix as input data, while AHP needs pairwise comparison matrix. Thus, when the DM can provide pairwise comparison matrix, then AHP will be the only left method to solve the decision making problem. AHP and its extended version Analytical Network Process (ANP) are implemented in *Super Decisions* software, which can be downloaded from *www.superdecisions.com*. Thus, only methodology instructions of AHP are integrated in the developed multi-criteria decision support system.

3.2.4 Step 4: Define the Preferences on Evaluation Criteria

Usually, after the initial screening step is completed, multiple MCDA methods are expected to remain, otherwise we can directly choose the only one left to solve the decision making problem. With the nine scoring questions defined in Step 2, the DM's preference information on the evaluation criteria is defined. This will reflect which criterion is more important to the DM in the method selection process.

In this study, relative weights are assigned to each evaluation criterion to describe the DM's preference information. The weights must be carefully considered based on the DM's preferences and experiences, we suggest using a subjective scale of 0 to 10 recommended by Hwang (39), with calibration that 0 stands for extremely unimportant while 10 represents extremely important. The procedures that derive these numerical values use addition and multiplication operations across attributes.

3.2.5 Step 5: Calculate the Appropriateness Index

In this study, sixteen widely used MCDA methods are identified and stored in the method database as candidate methods for selection. The evaluation criteria can be captured by answering three filter questions and nine scoring questions relevant to the characteristics of the methods. An Appropriateness Index (AI) (48), (77), as shown in

Equation 3.1, is used to rank the MCDA methods.

$$\begin{aligned} AI_j &= \sum_{i=1}^{n} w_i b_{ji} \\ b_{ji} &= \begin{cases} 1 \text{ if} c_{ji} = a_i \\ 0 \text{ if} c_{ji} \neq a_i \end{cases} \quad i = 1, 2, ..., n; j = 1, 2, ..., m \end{aligned} \tag{3.1}$$

where n is the number of evaluation criteria used to examine the methods with respect to the given problem, and m is the number of methods stored in the method library, $\{w_1, w_2, ..., w_n\}$ are the weighting factors for the evaluation criteria, a_i is the value of the i-th characteristic of the decision problem, and c_{ji} is the value of i-th characteristic of the j-th method, b_{ji} is a Boolean number depending on the match of the i-th characteristic of the decision problem and the i-th characteristic of the j-th method. If the i-th characteristic of the decision problem matches the i-th characteristic of the j-th method, then $b_{ji} = 1$; otherwise, $b_{ji} = 0$.

In this study, in order to better distinguish the appropriateness of the candidate MCDA methods, it is recommended that the decision rule (the fifth evaluation criterion) is the determinant evaluation criterion. Thus, if the characteristic of the decision problem matches the characteristic of the method regarding the fifth evaluation criterion, then the method obtains $5 \times b_{ji}$.

Table 3.1 shows one example of the AI calculation process for TOPSIS technique. At first, the DM identifies the key characteristics of the decision making problem by defining relative weights for the evaluation criteria. In this example, the decision rule, input data, and uncertainty analysis, are considered as most important criteria, so high weights are assigned to these evaluation criteria. The other evaluation criteria are assigned relative weights in the same way, thus, the weighting factors of the nine evaluation criteria are defined as [5 8 4 4 6 4 3 6 5]. Second, the characteristics of the decision making problem are obtained from the answers to the questionnaire, while the characteristics of the MCDA methods can be obtained from the method database. Then, the characteristics of the problem and method are compared pairwise in order to see if they match with each other. Finally, AI can be calculated for TOPSIS by using Equation 3.1 and the result is given by Equation 3.2.

Table 3.1: The Appropriateness Index Calculation Process for TOPSIS

Evaluation Criteria	Criteria Weights w_i	Problem criteria values a_i	Method criteria values c_{ji}	Match scores b_{ji}
Filter Questions				
1. Selection/Optimization	-	-	-	-
2. Allow trade-off	-	-	-	-
3. Input data	-	-	-	-
Scoring Questions				
4. Preference information	5	Relative weight	Relative weight	1
5. Decision rule	8	Min. Closeness	Min. Closeness	1
6. Feasibility evaluation	4	Yes	No	0
7. Subjective	4	No	No	1
8. Qualitative/quantitative data	6	Quantitative	Quantitative	1
9. Discrete/continuous data	4	Discrete	Discrete	1
10. Single/hierarchy	3	Single	Single	1
11. Capture uncertainties	6	Yes	No	0
12. Visualization	5	Yes	Yes	1

$$\begin{aligned} AI_{TOPSIS} &= \sum_{i=1}^{9} w_i b_{ji} \\ &= [5\ 8\ 4\ 4\ 6\ 4\ 3\ 6\ 5] * [1\ (1 \times \mathbf{5})\ 0\ 1\ 1\ 1\ 1\ 0\ 1]^{\mathrm{T}} \\ &= 67 \end{aligned} \tag{3.2}$$

As noted in the previous paragraph, considering the determinate role of the decision rule (the fifth evaluation criterion) in the method selection process, five times score is obtained if the fifth characteristic of the given problem match the fifth evaluation criterion of the method, with the purpose of better distinguishing the appropriateness of the MCDA methods.

3.2.6 Step 6: Evaluate the MCDA Methods

In order to compare the appropriateness of the methods with respect to the given decision making problem, each method is evaluated based on the nine scoring questions and AIs of the MCDA methods are obtained. Based on the AI calculation, the MCDA method with the highest score will be chosen as the most appropriate method to solve the original decision making problem.

3.2.7 Step 7: Choose the Most Suitable Method

As noted in Step 6, the method with the highest AI will be recommended as the most appropriate method to solve the given problem. The developed decision support system is utilized to guide the user to reach the final decision when solving evaluation decision making problems. After one MCDA method is identified as the most appropriate method, the user can simply click the name of the method, and the methodology instructions will be displayed to guide the user to solve the given problem. The detailed mathematical calculation steps are built in the MATLAB-based decision support system, thus, the user can just simply follow the instructions, such as inputting necessary data, to get the final results.

3.2.8 Step 8: Conduct Sensitivity Analysis

It is observed that different DMs often have different preference information on the evaluation criteria and different answers to the twelve evaluation questions, thus, sensitivity analysis should be performed on the method selection algorithm in order to analyze its robustness with respect to parameter variations, such as the variation of DM's preference information and the input data.

We strongly recommend that the DM should treat the weighting factors of each characteristic in a parametric manner. In our integrated user interface, the DM can adjust the weights of each criterion by moving the corresponding slide bars. If the DM is satisfied with the final results, the solution can be implemented. Otherwise, DM can go back to Step 2 and modify the input data or preference information and repeat the selection process until a satisfying outcome is obtained.

It is worth noting that there is no absolute *best* MCDA method since the MCDA method selection is problem specific. The selection of the most suitable MCDA method depends on the problem under consideration.

3.3 An Intelligent Multi-Criteria Decision Support System

The proposed approach to method selection is implemented and an intelligent multi-criteria decision support system is developed in MATLAB. The architecture of the intelligent multi-criteria decision support system is illustrated in Figure 3.2. For a given decision making problem, the DM needs to define the requirements of the problem and the preference information on these requirements. Then the intelligent multi-criteria decision support system will utilize the information provided in the knowledge base, and rank the methods stored in the method database. The method with the highest appropriateness score will be selected as the most appropriate MCDA method to solve the given decision making problem.

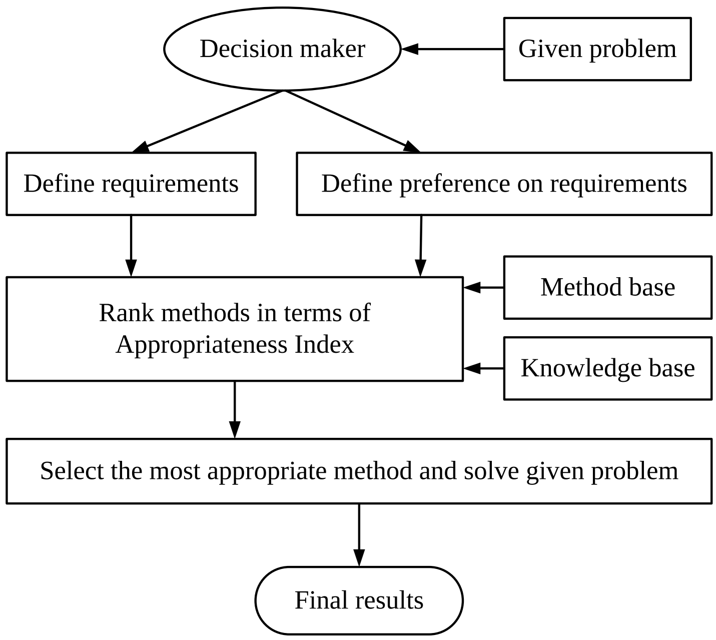


Figure 3.2: The Architecture of an Intelligent Multi-Criteria Decision Support System

The user guide of the intelligent knowledge-based decision support system developed in MATLAB Graphical User Interface (GUI) can be found in Appendix B.

3.4 Chapter Summary

An advanced approach to effectively select the most appropriate MCDA method for a given decision making problem was presented in this chapter. Twelve evaluation criteria were proposed to assess sixteen widely used MCDA methods. This method selection approach was implemented and an intelligent multi-criteria decision support system was developed in MATLAB. The capabilities of the developed intelligent multi-criteria decision support system will be demonstrated and evaluated in Chapter 5 and Chapter 6.

4

Uncertainty Assessment in the Decision Analysis Process

The second objective of this research is the assessment of uncertainties propagated in the decision analysis process when solving a decision making problem. The inherent uncertainties of the input data have significant impacts on the final decision solution. In this chapter, a new approach for uncertainty assessment in the decision analysis process is proposed. This approach consists of four steps: uncertainty characterization by percentage uncertainty with confidence level, uncertainty analysis using error propagation techniques, local sensitivity analysis based on an iterative binary search algorithm and global sensitivity analysis using partial rank correlation coefficients. Each step of the uncertainty assessment approach will be discussed in detail in the following sections.

4.1 Uncertainty Characterization

The uncertainties are described by percentage uncertainties with different confidence levels. These uncertainties are transferred into standard deviations through the utilization of inverse error function. In this section, the relationship between normal distribution and error function is introduced first, then the uncertainty transformation using inverse error function is presented.

4.1.1 Relationship Between Normal Distribution and Error Function

For a normal random variable X with $N\left(\mu, \sigma^2\right)$, the probability of a random sample value falling within the interval $[\mu - n\sigma, \mu + n\sigma]$ can be calculated by

$$\begin{aligned} P\left(\mu - n\sigma < X < \mu + n\sigma\right) &= \int_{\mu-n\sigma}^{\mu+n\sigma} f(x)dx \\ &= \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu-n\sigma}^{\mu+n\sigma} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)} dx \end{aligned} \tag{4.1}$$

The error function is shown in Equation 4.2 (62), with the substitution $z = \frac{X-\mu}{\sigma}$, Equation 4.1 can be converted into Equation 4.3

$$y = erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{\left(-t^2\right)} dt \tag{4.2}$$

$$\begin{aligned} P\left(\mu - n\sigma < X < \mu + n\sigma\right) &= \frac{1}{\sqrt{2\pi}} \int_{-n}^{n} e^{\left(-\frac{z^2}{2}\right)} dz \\ &= erf\left(\frac{n}{\sqrt{2}}\right) \end{aligned} \tag{4.3}$$

In other words, the probability of a normal random variable X falling within its interval $[\mu - n\sigma, \mu + n\sigma]$ can be calculated by the error function $erf\left(\frac{n}{\sqrt{2}}\right)$. Some typical numbers of standard deviation are plotted in Figure 4.1.

4.1.2 Uncertainty Transformation using Inverse Error Function

When the probability (confidence level) of a normal random variable X falling within certain confidence interval has been given, the corresponding numbers of standard deviation can be calculated by the inverse error function, as described in Equation 4.4.

$$n = \sqrt{2} erf^{-1}(\text{Confidence level}) \tag{4.4}$$

The relationship between mean μ and standard deviation σ is shown in Equation 4.5.

$$\text{Relative error}(\%)\mu = n\sigma \tag{4.5}$$

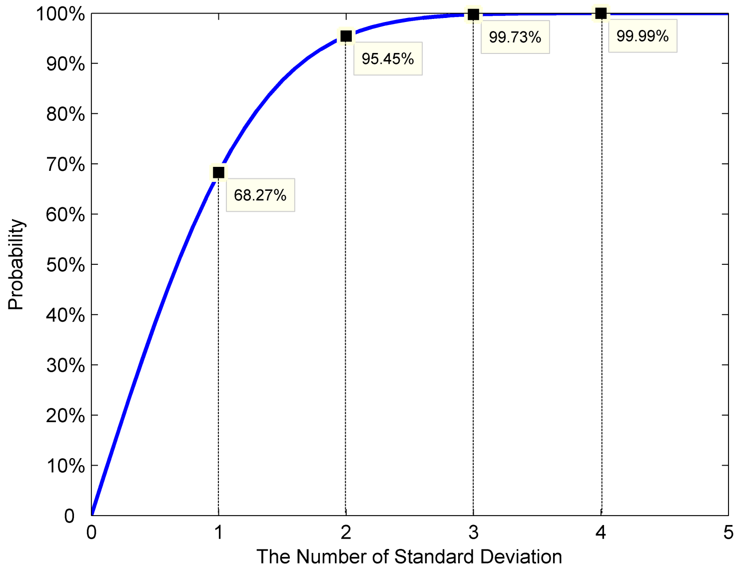


Figure 4.1: Typical Numbers of Standard Deviation

Note that relative error here is equivalent to percentage uncertainty, thus, the conversion of percentage uncertainty into standard deviation is shown in Equation 4.6.

$$\sigma = \frac{\text{Percentage uncertainty}(\%)\mu}{n} \tag{4.6}$$

In this step, we have already transferred percentage uncertainty with certain confidence level into its standard deviation. The uncertainties of the input parameters are expressed by their means and standard deviations, which will be the input data of the propagated error calculation.

4.2 Uncertainty Analysis

The process of uncertainty analysis using error propagation techniques is illustrated in Figure 4.2. In the first part of this section, the background of error propagation techniques is introduced. Robustness measurement using Signal-to-Noise Ratio (SNR) is presented in the second part.

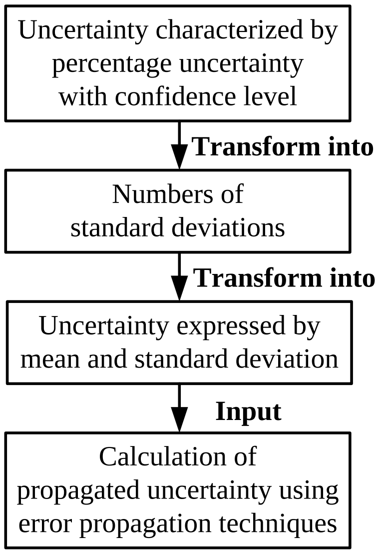


Figure 4.2: The Process of Uncertainty Analysis using Error Propagation Techniques

4.2.1 Background of Error Propagation Techniques

Error propagation techniques answer the question: how the uncertainties of input variables will be propagated to some predefined functions involving these variables and lead to the final result (10). There are two classes of error propagation techniques: analytical and simulation-based numerical error propagation techniques.

Analytical error propagation technique relies on a linearized Taylor series expansion of the function about the mean value of each variable, the total error of the function is obtained by combining the linearized individual error in quadrature. For a function

$$y = f(x_1, x_2, ..., x_n) \tag{4.7}$$

where $x_1, x_2, ..., x_n$ are input variables, $\delta_{x_1}, \delta_{x_2}, ..., \delta_{x_n}$ refer to the relatively small uncertainties in $x_1, x_2, ..., x_n$, respectively. The small uncertainties can be identified as Gaussian distribution provided that their magnitude is not too large (10). Small uncertainties of the variables $\delta_{x_1}, \delta_{x_2}, ..., \delta_{x_n}$ can be used with their standard deviation $\sigma_{x_1}, \sigma_{x_2}, ..., \sigma_{x_n}$ interchangeably. Based on Taylor series expansions, the propagated errors of input variables $x_1 \pm \delta_{x_1}, x_2 \pm \delta_{x_2}, ..., x_n \pm \delta_{x_n}$ can be analytically described by the error propagation Equation 4.8 (10).

$$\sigma_y^2 = \sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j}\right)^2 \sigma_{x_j}^2 + 2\sum_{j=1}^{n}\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_j}\right)\left(\frac{\partial f}{\partial x_i}\right) \sigma_{x_j x_i} (i \neq j) \tag{4.8}$$

where σ_y^2 is the total variance of the function, $\frac{\partial f}{\partial x_j}$ is a partial derivative of the function f with respect to variable x_j, when treating other variables $x_1, x_2, ..., x_{j-1}, x_{j+1}, ..., x_n$ as constants, $\sigma_{x_j}^2$ is the variance of variable x_j, $\sigma_{x_j x_i}^2$ is the cross-product covariance when variables x_j and x_i are correlated. If the variables $x_1, x_2, ..., x_n$ are independent, we can omit the cross-product covariance term, the error propagation Equation 4.8 reduces to

$$\sigma_y^2 = \sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j}\right)^2 \sigma_{x_j}^2 \tag{4.9}$$

The contribution due to the uncertainties in $x_1, x_2, ..., x_n$ is considered separately through Equation 4.9, provided that the errors of the input variables could be seen as normally distributed and there is no strong nonlinearity associated with the function in its evaluation range.

While analytical error propagation technique is appropriate for simple calculation processes, simulation-based numerical error propagation technique is more suitable for complex models, where trade-off has to be made between results accuracy and computation time.

One Example of Uncertainty Analysis for a Car Selection Problem

One example of uncertainty analysis for a car selection problem, as described in Subsection 2.2.4, is conducted in this subsection. The decision matrix summarized in Table 4.6 is repeated here for the convenience of calculation.

Table 4.1: The Decision Matrix of a Car Selection Problem for Uncertainty Analysis

	Criteria		
	C_1: Handling	C_2: Fuel-economy	C_3: Power
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
A_1: Ford	8	7	10
A_2: Lexus	9	6	5
A_3: Saab	6	7	8

Assume that the DM states that there are 15% uncertainties existing in the criteria values with 80% confidence level, and there are 30% uncertainties existing in the weighting factors with 90% confidence level. Following the proposed uncertainty analysis approach described above, these percentage uncertainties with confidence levels are transferred into mean values and standard deviations, then Monte Carlo-based numerical error propagation technique is used to calculated the propagated errors.

When SAW is used to solve the car selection problem, the probabilistic ranking of the three candidate cars is summarized in Table 4.2. The largest number in each row indicates the most likely ranking. It can be observed that A_1 (Ford) has the highest probability to be ranked first, A_2 (Lexus) has highest probability to be ranked in the second place, and A_3 (Saab) has the highest probability to be ranked in the last place.

Table 4.2: The Probabilistic Ranking in the Car Selection Example

	Alternatives		
Ranking	A_1	A_2	A_3
1st	78.00%	19.00%	3.00%
2nd	21.00%	52.00%	27.00%
3rd	1.00%	29.00%	70.00%

In addition to the probabilistic ranking of each alternative, the likelihood for alternatives permutation is also calculated and demonstrated in Figure 4.3, where the vertical axis represents all possible ranking permutations of alternatives, the lower horizontal axis stands for simulation runs, and the upper horizontal axis corresponds to occurrence percentage of each alternatives permutation. It can be seen from Figure 4.3 that the alternative permutation $[A_1 \;\; A_2 \;\; A_3]$ has the highest probability of occurrence.

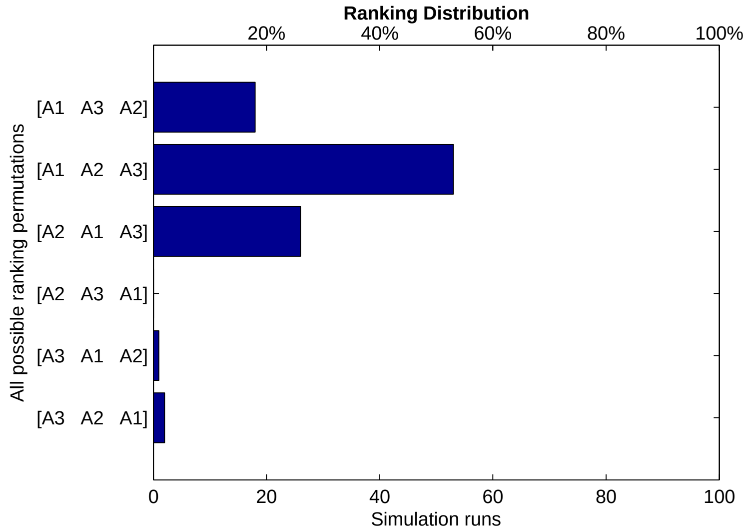


Figure 4.3: The Probabilistic Ranking Permutations in the Car Selection Example

4.2.2 Robustness Measurement using Signal-to-Noise Ratio

Robustness is an important performance measurement when uncertainty exists. Taguchi pioneered the application of robust design methods in the product design and manufacturing process (78). Robustness reflects product's ability to withstand uncontrollable variations in production and usage. The Signal-to-Noise Ratio (SNR) is one way to measure the robustness in Taguchi's method. The SNR in terms of mean and standard deviation is defined as Equation 4.10.

$$\mathrm{SNR} = 20\log_{10}(\frac{\mu}{\sigma}) \tag{4.10}$$

The SNR is expressed in decibel (dB). For instance, 40 (dB) means that the magnitude of mean is $10^{\frac{40}{20}} = 100$ times the magnitude of standard deviation. A larger SNR value indicates more robustness against uncertainty.

Moreover, linearity also influences the SNR value. When the relationship of the input and output of a system is not linear, deviation from linearity is taken as the error after the decomposition of variation and the SNR becomes smaller (78).

4.3 Local Sensitivity Analysis via Iterative Binary Search Algorithm

Sensitivity analysis addresses the question how does the variation of input variables influence model output (36). In general, there are two broad categories: local sensitivity analysis and global sensitivity analysis (72).

Local sensitivity analysis varies input variables one at a time to determine which variables have the greatest effect on the model output, while holding the others fixed at nominal values. Local sensitivity analysis has been widely used, given that the efficient computation of local sensitivities based on the variation of one variable at a time, and an initial understanding of the sensitivity of individual variable on model output over a small region around the nominal values of input variables can be obtained. However, local sensitivity analysis may not provide meaningful results when the model under consideration is nonlinear, or when input variables are perturbed simultaneously and by different amounts, and the effects of interactions among input variables on the model output cannot be captured (29), (61).

Global sensitivity analysis varies all variables simultaneously over the full range and investigates the influence of each variable averaged over all possible values of the other input variables (72), (29). Global sensitivity analysis can provide insights into model behavior over the full range of model outputs, taking into account the variable interactions (61). However, computational cost of global sensitivity analysis is higher than local sensitivity analysis and may become prohibitive for large complex models.

In this research, we take the perspectives that different types of sensitivity analysis reveal model behaviors in different domains of the variables (86), and global sensitivity analysis should not precede local sensitivity analysis (33). This subsection focuses on local sensitivity analysis when solving evaluation decision making problems, and global sensitivity analysis will be investigated in the next subsection.

Local Sensitivity Analysis in the Decision Analysis Process

When the MCDA methods are utilized in evaluation decision making problems, local sensitivity analysis can be conducted to determine the sensitivity of the rankings of the alternatives to changes in the input variables. A unified local sensitivity analysis approach for three MCDA methods including SAW, multiplicative weighting method, and AHP, was proposed (81), where two questions were addressed: (1) How sensitive the ranking of the best alternative or any alternative is to variations in the current weights or performance measures of decision criteria? (2) What is the smallest change in the current weights or performance measures of decision criteria which can alter the current ranking of two alternatives?

However, the proposed sensitivity analysis approach is specific for these three MCDA methods. When other MCDA methods are chosen to solve a given problem, the proposed sensitivity analysis approach is not applicable. In addition, this sensitivity analysis approach was obtained through the analytical inferences of these three specified MCDA methods, which only involve limited simple mathematical calculation steps. For instance, SAW just has two simple calculation steps: multiplication and addition, multiplicative weighting method only has two times multiplication, and AHP also merely involves multiplication and addition. Since only limited simple calculation steps are involved, it is easy to perform sensitivity analysis analytically. Nonetheless, for other widely used MCDA methods which need complicated mathematical calculations, such as TOPSIS or ELECTRE, it is difficult to infer the sensitivity coefficient for each input variable analytically. Thus, this sensitivity analysis approach cannot be extended for general MCDA methods.

In this study, an iterative binary search algorithm is developed to investigate how sensitive the ranking of the alternatives to variations in the weights or performance values of decision criteria. The iterative binary search algorithm can overcome these drawbacks mentioned above, since it is a sampling-based method which will not be affected by the analytical calculation steps of MCDA methods. Additionally, it can be generalized to other MCDA methods.

4.3.1 Iterative Binary Search Algorithm

The binary search technique has been widely used to find a target value in a sorted (usually ascending) sequence efficiently (83), (56). This technique compares the middle element of the sorted sequence to the target value, if the middle element is equal with the target value, then the search terminates. If the target value is less than middle element, then the algorithm eliminates the right half of the sorted sequence and conducts the same search for the left side. If the target value is bigger than the middle element, then the algorithm ignores the left half of the sorted sequence and performs the same search for the right side. Otherwise, we can conclude that the target value is not in the sorted sequence.

For example, given a sorted sequence [0 5 12 17 23 25 50 60 80], assume that we want to find the target value 25. The binary search technique works as follows.

- First iteration: [0 5 12 17 **23** 25 50 60 80]. The target value 25 is bigger than the middle element 23, ignore the left half of the sorted sequence, and perform the same search for the right side.
- Second iteration: [25 **50** 60 80]. The target value 25 is smaller than the middle element 50, ignore the right side of the sorted sequence, and perform the same search for the left side.
- Third iteration: [**25**] . The target value 25 is equal to the element 25, the target value is found.

When using the MCDA methods to solve given decision making problem, the input parameters are the values of decision criteria, the weighting factors, the original ranking of the alternatives, and the number of iterations. The outputs of the iterative search algorithm are the minimum changes in the values of decision criteria and the minimum changes in the weighting factors in order to alter the rankings of two alternatives. The iterative binary search algorithm varies one input variable at a time in order to find the minimum change in this input variable, which can alter the ranking of two alternatives.

In the iterative search algorithm, the first step is to initialize input parameters: left lower bound `ll_bound`, left upper bound `lu_bound`, right lower bound `rl_bound`, and right upper bound `ru_bound`. In the next step, the left trial value `l_trial` is calculated by the middle element of the left search space `(ll_bound + lu_bound)/2`, and the right trial value `r_trial` is calculated by the middle element of the right search space `(rl_bound + ru_bound)/2`, as illustrated in Figure 4.4.

The flow chart of the iterative binary search algorithm is shown in Figure 4.5, where `l` stands for left and `u` upper, `ll` stands for left lower, `lu` left upper, `rl` right lower, and `ru` right upper. `delta` is the minimum change in weights or decision criteria when two rankings are altered. The default setting is that it is non-feasible to change the current parameter to alter the ranking of the two alternatives. The iteration `runs` determines

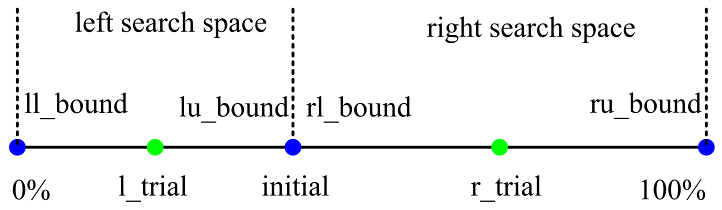


Figure 4.4: Initialization for the Iterative Binary Search Algorithm

the precision of the calculation $2^{\texttt{runs}}$ (56). For instance, when the iteration runs is set as `runs` $= 30$, the precision of the calculation is $2^{\texttt{runs}} = 2^{30} = 1.0737e+009$. The initial runs is set as $i = 0$.

The new trial values of the parameter under consideration are calculated and new rankings of alternatives are computed. The rankings in the left search space will be evaluated first. If the rankings using left new trial value change, then we will assign `true` to the judgment variable `isFeasible`, and calculate the relative quantity of the parameter under consideration `delta_decrement`, and the left new trial value `l_trial` is assigned to the left lower bound `ll_bound`. If the ranking using left new trial value does not change, then, the left new trial value `l_trial` is given to the left upper bound `lu_bound`. After the evaluation of the left search space, the similar procedure is performed to the right search space. The algorithm is terminated when the iteration runs is finished. Finally, if the judgment variable `isFeasible` is `true`, the absolute magnitude of the relative quantities `delta_decrement` and `delta_increment` is compared. The smaller quantity `delta` is the minimum change which can alter the rankings of two alternatives. Otherwise, we can conclude that it is not feasible to change the current parameter so that the rankings of two alternatives is altered.

Since the binary search technique is used for a sorted sequence, attention should be paid on the monotonicity of MCDA methods. If the MCDA methods are non-monotonic, the iterative search algorithm may generate misleading results.

4.3.2 Interactive Sensitivity Analysis for Weighting Factors

It is observed that the weighting factors are often highly subjective considering the fact that they are elicited based on DM's experience or intuition. The inherent uncertainties and subjectivities of the weighting factors have significant impacts on the final result of a decision making problem. This implies that it is critical to effectively address these uncertainties in the decision making process in order to get more accurate results.

In this study, an interactive sensitivity analysis is developed with the purpose of providing more informed decision aiding. The basic idea of the interactive sensitivity analysis of weights is to vary the weight of one criterion from 0 to 100%, while keeping the weights of other criteria the same proportion as in the original setting.

One Example of Local Sensitivity Analysis for a Car Selection Problem

One example of local sensitivity analysis for a car selection problem, as described in Subsection 2.2.4, is conducted in this subsection. The decision matrix shown in Table 4.6 is repeated here for the convenience of calculation.

Table 4.3: Decision Matrix of a Car Selection Problem for Local Sensitivity Analysis

	Criteria		
	C_1: Handling	C_2: Fuel-economy	C_3: Power
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
A_1: Ford	8	7	10
A_2: Lexus	9	6	5
A_3: Saab	6	7	8

When SAW is used to solve the car selection problem, the ranking of the three alternatives is $[A_1 \quad A_2 \quad A_3]$. The developed iterative binary search algorithm can answer the question: What is the smallest change in the weighting factors so that the ranking of the most preferred alternative or any alternative will be altered?

The absolute minimum changes in the weighting factors which can alter the ranking of the alternatives are summarized in Table 4.4. For the convenience of comparison, the relative minimum changes in the weighting factors which can alter the ranking of the alternatives are also presented in Table 4.5. The relative minimum changes are the absolute minimum changes scaled against the original values of the weighting factors. In these two tables, N/F (Non-Feasible) means that it is not mathematically feasible to alter the ranking of the alternatives through the change of the current parameter.

The first two rows in Table 4.5 show that when the weighting factor of C_3 de-

Table 4.4: Absolute Minimum Changes in Weighting Factors to Alter the Rankings of Alternatives in the Car Selection Example

Pairs of Rankings	C_1	C_2	C_3
A_1:A_2	0.54	0.42	-0.12
A_1:A_3	N/F	N/F	N/F
A_2:A_3	-0.21	N/F	0.23

Table 4.5: Relative Minimum Changes in Weighting Factors to Alter the Rankings of Alternatives in the Car Selection Example

Pairs of Rankings	C_1	C_2	C_3
A_1:A_2	178.58%	104.17%	-39.69%
A_1:A_3	N/F	N/F	N/F
A_2:A_3	-67.15%	N/F	74.61%

creases -39.69%, A_2 (Lexus) becomes the most preferred alternative, and it is not possible to change the weighting factors so that A_3 (Saab) ranks first. Moreover, it can be seen from the whole table that the weighting factor of C_3 is most sensitive to the ranking of the three alternatives.

Furthermore, following the proposed idea of varying the weighting factor of one criterion from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting, the interactive sensitivity analysis of the weighting factor of C_1 is illustrated as an example in Figure 4.6, where the intersection of the lines indicate that there is ranking change between the alternatives.

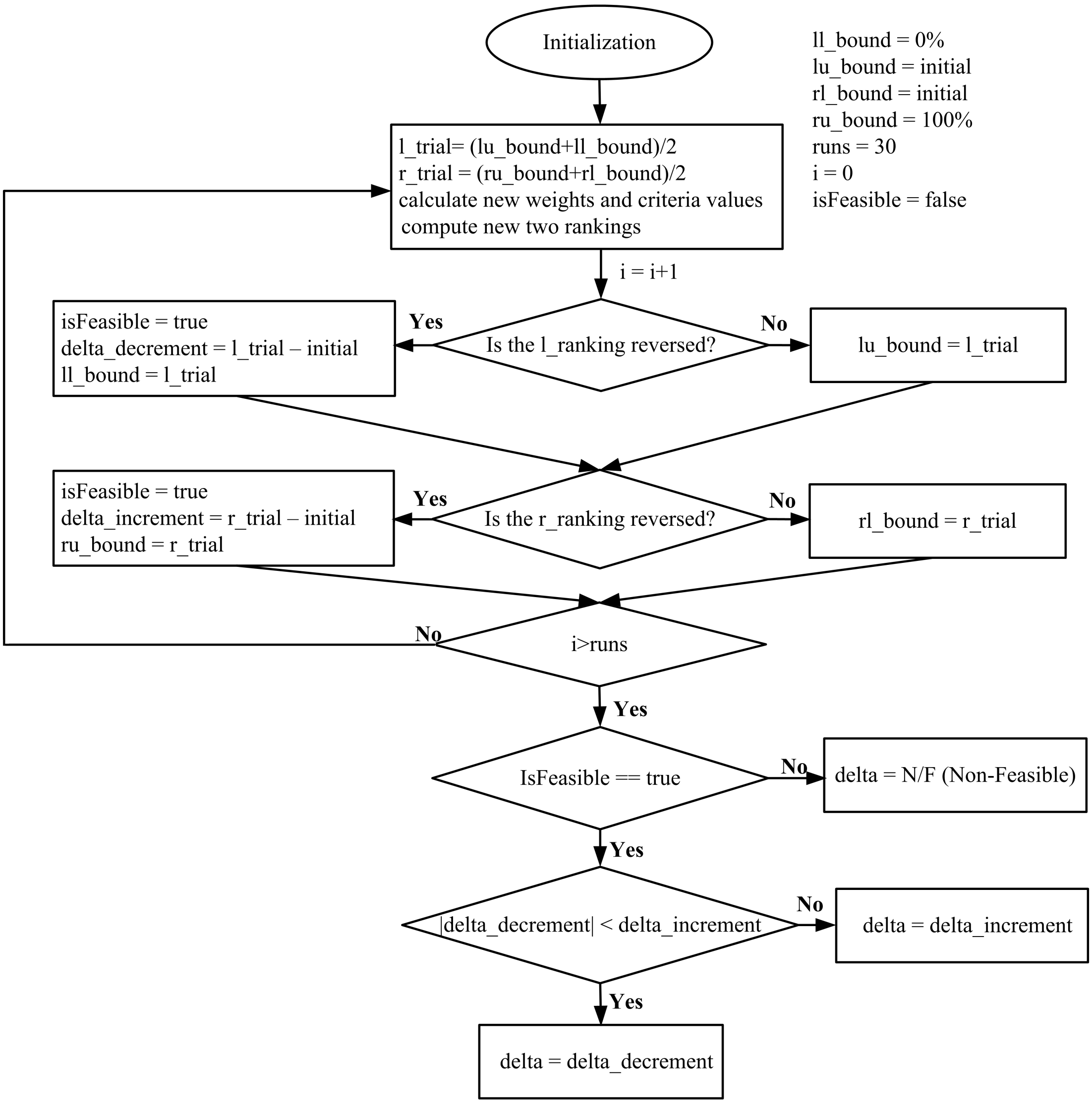


Figure 4.5: Flow Chart of the Iterative Binary Search Algorithm

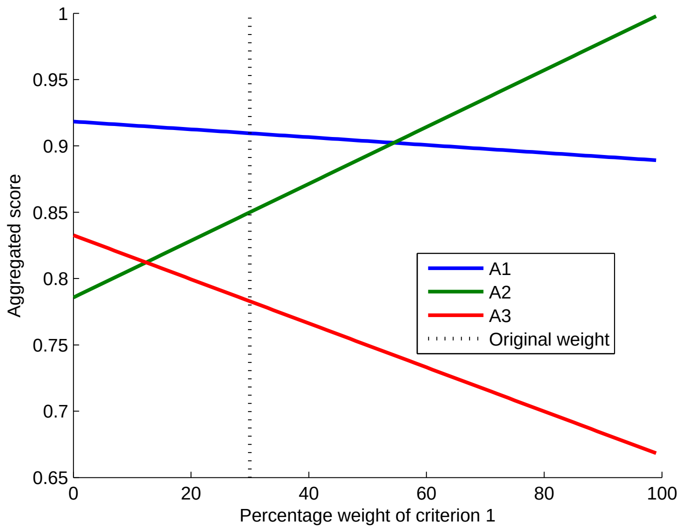


Figure 4.6: Interactive Sensitivity Analysis for the Weighting Factor of C_1 in the Car Selection Example

4.4 Global Sensitivity Analysis Using Partial Rank Correlation Coefficients

In contrast to local sensitivity analysis, global sensitivity analysis allows the variations of all variables over the full range at the same time. Many techniques have been developed to perform global sensitivity analysis, among which Monte-Carlo sampling and correlation analysis (11), (54), (36) and variance decomposition analysis (72) are two most popular methods.

In this research, considering that the existences of inherent uncertainties in the decision analysis process, especially the subjectivities of the weighting factors, have significant impacts on the final result of a decision making problem, statistical techniques are capable of effectively dealing with these uncertainties. Therefore, global sensitivity analysis based on Monte-Carlo sampling and correlation analysis will be further investigated.

4.4.1 Correlation Coefficients and Statistical Significance Test

In the decision analysis process, decision criteria and preference information are the main input variables utilized to solve the evaluation decision making problem. The output variables of the MCDA model are the overall performances of alternatives, which can be illustrated by the preferred ranking of the candidate alternatives or the classification of the outperformed set (7),(28). The input variables and output variables in MCDA models for statistical analysis are illustrated in Figure 4.7.

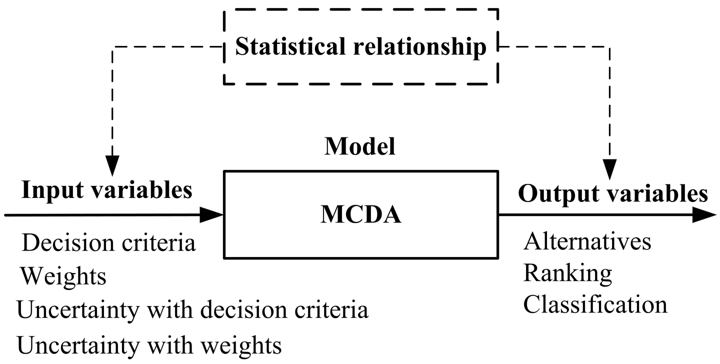


Figure 4.7: The Input Variables and Output Variables in the Decision Analysis Process

The degree of association is one way to describe the statistical relationship between input variables and output variables in the decision analysis process. Association between two variables exists when knowing the value of one variable provides information about the likely value of the other variable, while correlation between the two variables exists when the association is linear (37). There are several correlation coefficients measuring the degree of association: Pearson correlation coefficient, Spearman rank correlation coefficient, and partial rank correlation coefficient (74). The following part of this subsection introduces these three correlation coefficients and statistical significance test.

Pearson Correlation Coefficient

Pearson correlation coefficient r is one of the most common measures of linear relationship between two variables. Without loss of generality, assume that two variables X and Y, with sample values $x_1, x_2, ..., x_n$ and $y_1, y_2, ..., y_n$, are well approximated by normal distributions, and their joint probability distribution is a bivariate normal distribution. Pearson correlation coefficient is calculated by

$$\begin{aligned} r &= \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{var}(X)}\sqrt{\mathrm{var}(Y)}} \\ &= \frac{\sum_{i=1}^{n} \frac{(x_i-\bar{x})(y_i-\bar{y})}{n}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n}}} \\ &= \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}} \end{aligned} \tag{4.11}$$

where *cov* represents the covariance of two variables, *var* represents the variance of one variable, $\bar{x}$ is the mean of the sample values $x_1, x_2, ..., x_n$, and $\bar{y}$ is the mean of the sample values $y_1, y_2, ..., y_n$.

Pearson correlation coefficient r ranges from -1 to +1. A value of -1 indicates a perfect negative linear relationship between variables X and Y, a value of +1 implies a perfect positive linear relationship between variables X and Y, a value of 0 shows that there is no linear correlation between variables X and Y.

Spearman Rank Correlation Coefficient

Spearman rank correlation coefficient r_s is a non-parametric measure of association between two variables, which are measured in ordinal scale, without the assumption that the variables are normally distributed. When the association between X and Y is nonlinear, the relationship can be transferred into a linear one by using the ranking of the values of the variables R_{x_i} and R_{y_i} rather than their actual values. If there are no tied ranks, the result of Equation 4.11 with rank transformed variables is called Spearman rank correlation coefficient. Spearman rank correlation coefficient can also be calculated by Equation 4.12 (46)

$$r_s = 1 - \frac{6\sum\limits_{i=1}^{n}\left(R_{x_i} - R_{y_i}\right)^2}{n\left(n^2-1\right)} \tag{4.12}$$

If tied ranks occur, the same rank has to be assigned to the equal values, which is the average of their positions in the ascending order of the values, the correction factor T is defined as in Equation 4.13

$$T = \frac{t^3 - t}{12} \tag{4.13}$$

Then, Spearman correlation coefficient between ranks with the correction factors is calculated by Equation 4.14

$$r_s = \frac{\frac{n\left(n^2-1\right)}{6} - \sum\limits_{i=1}^{n}\left(R_{x_i} - R_{y_i}\right)^2 - \sum\limits_{i=1}^{n} T_x - \sum\limits_{i=1}^{n} T_y}{2\sqrt{\frac{n\left(n^2-1\right)}{12} - \sum\limits_{i=1}^{n} T_x}\sqrt{\frac{n\left(n^2-1\right)}{12} - \sum\limits_{i=1}^{n} T_y}} \tag{4.14}$$

Spearman rank correlation coefficient ranges from -1 to +1. A value of -1 indicates a perfect negative correlation between the two ranked variables, a value of +1 implies a perfect positive correlation between the two ranked variables, a value of 0 shows that there is no correlation between the two ranked variables.

Partial Rank Correlation Coefficient

Partial correlation coefficient measures the linear but monotonic association between two variables, if they were not each correlated with any other variables (51). Alternatively speaking, partial correlation coefficients determine what the association between any two of the variables, while eliminating indirect associations due to other variables (74). Without loss of generality, assume three variables X, Y, and Z, with sample values $x_1, x_2, ..., x_n$, $y_1, y_2, ..., y_n$, and $z_1, z_2, ..., z_n$. The partial correlation coefficient between X and Y, when eliminating the indirect associations due to relationships that may exist between X and Z or Y and Z, equals to Pearson correlation coefficient between the two residuals $X - \hat{X}$ and $Y - \hat{Y}$, as given by Equation 4.15.

$$r_{XY.Z} = \frac{r_{XY} - r_{XZ}r_{YZ}}{\sqrt{\left(1 - r_{XZ}^2\right)\left(1 - r_{YZ}^2\right)}} \tag{4.15}$$

where $\hat{X}$ and $\hat{Y}$ are based on the linear regression between X, Y and Z, as shown in Equation 4.16.

$$\begin{aligned} \hat{X} &= a_0 + a_1 Z \\ \hat{Y} &= b_0 + b_1 Z \end{aligned} \tag{4.16}$$

Partial rank correlation coefficient r_p calculates the partial correlation coefficient for the rank-transformed variables, which characterizes the linear but monotonic relationship between the rankings of the two variables while eliminating indirect associations due to other variables. Partial rank correlation coefficient varies between -1 and +1, where -1 represents strongest negative association and +1 represents strongest positive association between the input variables and model outputs.

Statistical Significance Test

The degree of association itself cannot uncover the relationship between the criteria and the rankings of alternatives without the statistical significance test. A strong association is not necessarily statistically significant (69), the interpretation of the association could be misleading without the statistical significance test. Therefore, it is crucial to conduct the measure of association and the statistical significance test in order to avoid improper decisions (32).

Hypothesis testing can be performed to evaluate whether the measure of association between two variables is statistically significant or not, which involves the calculation of a test statistic based on a random sample from the population to determine whether to reject a given hypothesis (62).

In addition, p-value provides another way to assess the statistical significance of the test statistic (62). The p-value is the probability value that the test statistic is at least as large as the observed one, given that the null hypothesis H_0 is true. The lower p-value provides stronger evidence to reject the null hypothesis H_0 in favor of the alternative hypothesis H_1.

4.4.2 Proposed Approach to Perform Global Sensitivity Analysis

Partial rank correlation coefficient is one of the popular sampling-based global sensitivity analysis indexes, it has been widely used to infer biochemical interactions in systems biology (11),(54). In the decision analysis process, partial rank correlation coefficient can be utilized to determine the global sensitivity of the ranking or classification of the alternatives to the input variables. A higher absolute partial rank correlation coefficient of the input variables shows larger impact on the ranking or classification of the alternatives.

In this study, global sensitivity analysis using partial rank correlation coefficients in the decision analysis process is performed, according to a step by step approach emphasized on the measure of association together with the statistical significance test. The proposed step by step approach is presented as follows.

Step 1: Define Probability Distributions for Input Variables

In the decision analysis process, input variables are the values of decision criteria and weighting factors to reflect DM's preference information. When the amount of available data is not sufficient to construct probability distribution functions, uniform or normal distributions are two popular alternatives for probability distribution functions. In a given problem, physical constraints of the decision criteria usually serve as the range of variable variation, while the weighting factors range from 0 to 1.

Step 2: Perform Latin Hypercube Sampling

Latin Hypercube Sampling (LHS) is a type of stratified Monte-Carlo sampling technique (55), where the distributions of input variables are divided into N equal probability intervals and the value of each input variable is then randomly sampled. The entire range for each variable is explored in a way that each value of each variable is used exactly once. LHS has the advantage that it requires fewer samples than simple random sampling to achieve the same accuracy (55). The efficiency of LHS enables to vary all variables at the same time with low computational cost in global sensitivity analysis.

The minimum value of sample size N for LHS is $\frac{3}{4}k$, where k is the number of input variables that are varied (11). However, it is not necessary that the result is better when a larger sample size is used. In addition to higher computational costs, larger sample size can make very weak relationship become significant. The significance of a weak relationship is not necessarily important in real-world applications (57).

Step 3: Rank Transformation for both Input Variables and MCDA Output

For each combination of the sampled values of the decision criteria and weighting factors, MCDA methods are utilized to calculate the overall performances of the alternatives. The input variables (decision criteria and weighting factors) and MCDA output (alternatives' performances) are transformed into ranks. For the convenience of calculation, the ranks are in ascending order of the original values. Although the ascending order seems contrary against the ranking of alternatives, it does not influence the calculation results of partial rank correlation coefficients, since both the input and output are transformed into ranks in a consistent manner.

For the scoring MCDA methods, it is straightforward to transform the scores into ranks in ascending order. Regarding tied ranks, the average rank is used instead. For example, for a score vector S

$$S = [0.01 \;\; 0.02 \;\; 0.03 \;\; 0.05 \;\; 0.02\,]$$

Counting from smallest to largest, 0.01 ranks first, the two 0.02 ranks second and third, thus, the average rank $(2+3)/2 = 2.5$ is used for both of them. The transformed ranks in ascending order of the score vector S_R are shown as follows.

$$S_R = [1 \;\; 2.5 \;\; 4 \;\; 5 \;\; 2.5\,]$$

For the classification MCDA methods, for instance, ELECTRE, the outrank set is assigned scores first as follows: the non-dominated alternatives are assigned score 1, while the dominated alternatives are assigned score 0. Next, the outrank set with scores is transformed into ranks as the scoring MCDA methods, in a similar way but with tied ranks. For example, considering five alternatives (A_1, A_2, A_3, A_4, A_5), where A_1, A_3, and A_4 are non-dominated alternatives, while A_2 and A_5 are dominated alternatives. In the first step, A_1, A_3, and A_4 are assigned score 1, while A_2 and A_5 are assigned score 0. Thus, the assigned score vector for the five alternatives S_A

$$S_A = [1 \ \ 0 \ \ 1 \ \ 1 \ \ 0\,]$$

Next, the assigned score vector S_A with tied values is transformed into ranks. Counting from smallest to largest, the two 0 rank first and second, then the average rank is $(1+2)/2 = 1.5$. The three 1 rank third, fourth and fifth, their average rank is $(3+4+5)/3 = 4$. The transformed ranks of the outrank set in ELECTRE are shown in S_{AR}

$$S_{AR} = [4 \ \ 1.5 \ \ 4 \ \ 4 \ \ 1.5\,]$$

Attention should be paid that too many tied ranks may reduce the statistical power of partial rank correlation coefficients. This will be approved in Chapter 6.

Step 4: Calculate Partial Rank Correlation Coefficients

With the rank-transformed data, partial rank correlation coefficients can be calculated. The partial rank correlation coefficients in global sensitivity analysis are used to characterize the nonlinear but monotonic statistical relationship between input variables and model outputs (11). Thus, it is recommended that before initiating the global sensitivity analysis, it is necessary to examine the scatter plots in order to detect the nonlinearities and non-monotonicities between input variables and model outputs.

Step 5: Conduct Statistical Significance Test

The measure of association alone cannot uncover the statistical relationship between variables without the statistical significance test. In the study, the p-value will be computed to assess the statistical significance of the partial rank correlation coefficient. A lower p-value provides stronger evidence to reject the null hypothesis H_0 (there is no partial correlation) in favor of the alternative hypothesis H_1 (there is nonzero partial correlation between the rank transformed variables).

Step 6: Results Interpretation

It is crucial to interpret the partial rank correlation coefficients together with the statistical significance test. Usually, p-values less than 0.05 indicate that the partial rank correlation coefficients are statistically significant. The application of partial rank correlation coefficient can offer the DM more insights into the relative contribution of the input variables to the total performances of the alternatives explicitly.

It is important to note that there are two components in a global sensitivity coefficient: the range of the input variable and the sensitivity coefficient of the output to this input variable (61). An input variable is identified as important in global sensitivity analysis if it has a wider range and larger sensitivity coefficient. On the contrary, an input variable is not identified as important in global sensitivity analysis if it has a narrow range, or if has a small sensitivity coefficient.

One Example of Global Sensitivity Analysis for a Car Selection Problem

One example of global sensitivity analysis for a car selection problem, as described in Subsection 2.2.4, is conducted in this subsection. The decision matrix shown in Table 4.6 is repeated here for the convenience of calculation.

Table 4.6: The Decision Matrix of a Car Selection Problem for Global Sensitivity Analysis

	Criteria		
	C_1: Handling	C_2: Fuel-economy	C_3: Power
Alternatives	w_1: 0.3	w_2: 0.4	w_3: 0.3
A_1: Ford	8	7	10
A_2: Lexus	9	6	5
A_3: Saab	6	7	8

When SAW is used to solve the car selection problem, the ranking of the alternatives is $[A_1 \;\; A_2 \;\; A_3]$. The proposed approach for global sensitivity analysis is performed, with emphasis on the measure of association together with the statistical significance test. The partial rank correlation coefficients with p-values for A_1 is illustrated in Figure 4.8, where the horizontal coordinate represents the partial rank correlation coefficients, and the vertical coordinate stands for the six input variables for A_1, including three criteria values and the weighting factors.

The corresponding p-values for partial rank correlation coefficients are next to the bars. Lower p-values provide stronger evidence of statistical significance. In this car selection example, p-values less than 0.05 indicate that the partial rank correlation coefficients are statistically significant.

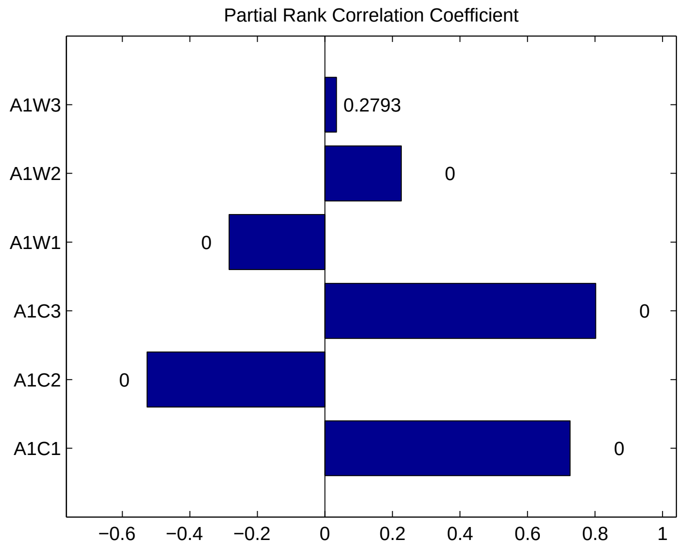


Figure 4.8: Partial Rank Correlation Coefficients for A_1 in the Car Selection Example

It can be observed from Figure 4.8 that C_3 shows the strongest statistically significant correlations with the overall performance of A_1, followed by C_1 and C_2.

4.5 An Uncertainty Assessment Module

The proposed new approach is implemented and an uncertainty assessment module is integrated with the developed intelligent multi-criteria decision support system in MATLAB, as shown in Figure 4.9. The user guide of the uncertainty assessment module can be found in Appendix B. In the uncertainty assessment module, the DM can simply go through the uncertainty assessment process according to the instructions. In addition, the mathematical calculation steps for four MCDA techniques: SAW, multiplicative weighting method, TOPSIS, and ELECTRE, are also built in the uncertainty assessment module, which highly facilitates the uncertainty assessment in the decision analysis process.

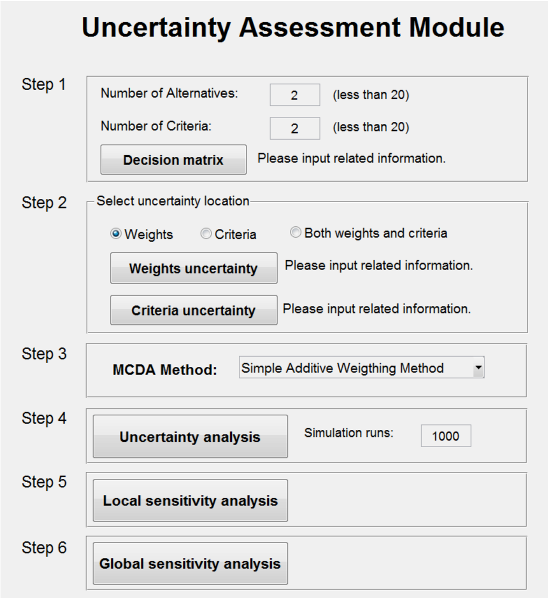


Figure 4.9: The User Interface of the Uncertainty Assessment Module

4.6 Chapter Summary

A new approach for uncertainty assessment in the decision analysis process was proposed in this chapter. This approach consists of four steps: uncertainty characterization, uncertainty analysis, local sensitivity analysis, and global sensitivity analysis. The proposed approach was implemented and an uncertainty assessment module was integrated with the developed intelligent multi-criteria decision support system, as discussed in Chapter 3.

Furthermore, a step by step approach to perform global sensitivity analysis using partial rank correlation coefficients was proposed in the study, with emphasis on the measure of association and statistical significance test. The proposed approach can be extended to investigate the statistical relationships between variables in complex analysis problems.

5

Proof of Concept 1: MCDA in Aircraft Design

The third objective of this research is to demonstrate the application of appropriate MCDA techniques with uncertainty assessment in aircraft design and aircraft evaluation decision making processes. In this chapter, the feasibility and added values of applying MCDA techniques in aircraft design are explored. A new optimization framework incorporating MCDA techniques in aircraft conceptual design process is established, as illustrated in Figure 5.1. An improved MCDA method is utilized to aggregate the multiple design criteria into one composite figure of merit, which serves as an objective function in the optimization process. The proposed optimization framework can support designers to quickly assess the compromised design alternatives, which is valuable especially in aircraft conceptual design stage.

The chapter is organized as follows. Section 5.1 defines the aircraft design problem. Section 5.2 presents the selection of the most appropriate MCDA method, through the intelligent multi-criteria decision support system, as described in Chapter 3. Section 5.3 presents the results of applying an improved MCDA method in the proposed multi-criteria optimization framework. In Section 5.4, surrogate model development for design criteria in terms of weighting factors is discussed. Section 5.5 presents uncertainty assessment based on the developed surrogate models, following the new approach proposed in Chapter 4. Section 5.6 discusses the implementation of MCDA techniques in aircraft design problems.

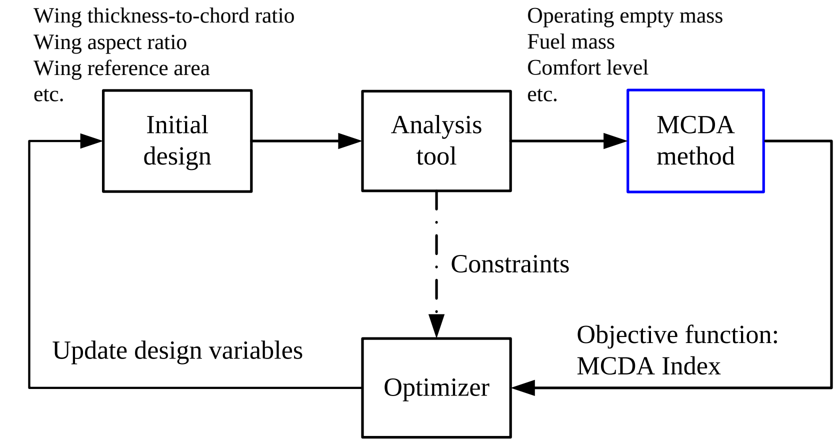


Figure 5.1: The Framework of Incorporating MCDA Techniques in Aircraft Design

5.1 Definition of the Decision Making Problem

The design of an A320-like commercial airliner is implemented as a proof of concept with the aircraft conceptual design tool VAMPzero (Virtual Aircraft Multidisciplinary Analysis and Design Processes) (12). VAMPzero is developed at German Aerospace Center (DLR e.V.) and licensed under the Apache 2.0 license. The design has 150 passenger, twin engine with 3200 km range. The simplified mission profile is illustrated in Figure 5.2.

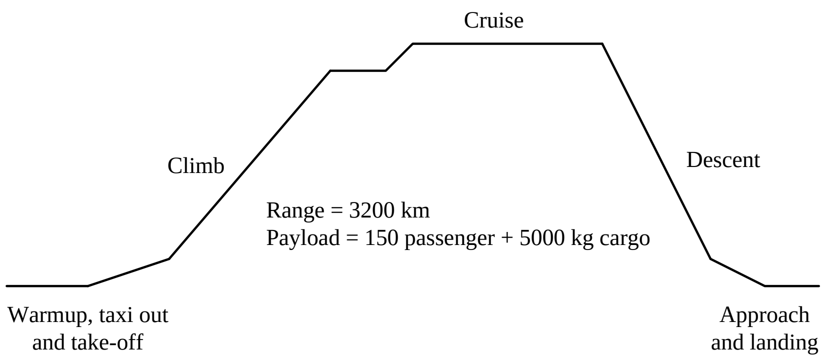


Figure 5.2: The Simplified Aircraft Mission Profile

The optimization framework shown in Figure 5.1 focuses on the assessment of added values of incorporating MCDA techniques in aircraft conceptual design process. Thus, in order to keep the design process transparent, the complexity of the design problem is limited. Five design variables for a conceptual aircraft design model are considered in this study: wing thickness-to-chord ratio, wing aspect ratio, wing reference area, cruise Mach number, and fuselage diameter. The baseline, minimum, and maximum values for the five design variables are listed in Table 5.1.

Table 5.1: The Baseline and Ranges of Design Variables

	Thickness-to-chord ratio	Aspect ratio	Reference area (m^2)	Cruise Mach number	Fuselage diameter (m)
Baseline	0.13	9.396	122.4	0.78	4
Minimum values	0.1	8	80	0.7	3.8
Maximum values	0.2	12	140	0.84	4.2

5.1.1 Identification of Design Criteria

The design criteria of interest are categorized into four groups: cost-based, weight-based, operation-based, and comfort-based. The four groups are described as follows.

Cost-based criteria

- **DOC**: DOC calculates all the direct operating costs per block hour, including fuel cost, maintenance cost, depreciation cost, crew cost, and miscellaneous cost.

- **Fuel cost**: Fuel cost calculates the mission fuel costs per block hour, as shown in Equation 5.1. Fuel price is set to 0.85 Dollars per kilogram.

- **Aircraft price**: An estimation of aircraft price based on OEM, is shown in Equation 5.2 (41). The exchange rate from Dollar to Euro is set to 0.73.

Weight-based criteria

- **OEM**: Operating Empty Mass (OEM) calculates the operating empty mass from the components, including fuselage, wing, engine, landing gear, horizontal tail plane, vertical tail plane, and pylon, and operator's items mass.

- **Fuel mass**: Fuel mass calculates the fuel needed for the complete mission via the sum of all mission segment fuel masses, including take-off, climb, cruise, descent, and reserve.

- **TOM**: Take-off Mass (TOM) is the sum of OEM, fuel mass, and payload.

Operation-based criteria

- **Annual Utilization**: The annual utilization defines the number of flight hours relative to the number of possible flight hours, with the assumption that the aircraft is grounded for a quarter of an hour. Its formula is shown in Equation 5.3 (35).

- **Block time**: Block time calculates the time from engines *on* to engines *off* for the design mission (41). The utilization/(block time) ratio provides the number of flight, as shown in Equation 5.4.

Comfort-based criteria

- **Passenger density**: Passenger density is defined by the number of passenger seats divided by cabin base area, where cabin base area is calculated by the product of fuselage diameter and cabin length. Its mathematical formula is shown in Equation 5.5.

$$\text{Fuel Cost} = \left(\frac{\text{Fuel mass} \times \text{Fuel price}}{\text{Block time}}\right)(\text{Exchange rate}) \tag{5.1}$$

$$\text{A/C Price} = (0.8109\left(\frac{\text{OEM}}{1000}\right) + 6.3722)(\text{Exchange rate})(\text{Inflation rate})10^6 \tag{5.2}$$

$$\text{Annual Utilization} = \frac{4198}{1 + \frac{0.75}{\text{Block time}}} \tag{5.3}$$

$$\text{Utilization/(Block time)} = \frac{4198}{0.75 + \text{Block time}} \tag{5.4}$$

$$\text{Passenger Density} = \frac{\text{Number of passenger seats}}{\text{Fuselage diameter} \times \text{Cabin length}} \tag{5.5}$$

Selection of appropriate design criteria is critical to the determination of an optimal design. Some recommendations were provided in (65): the design criterion should represent a non-trivial and calculable indication of the worth of the concept, it should be significantly affected by the design variables and constraints, it should have clear meaning to designers and customers, and it needs clear rationale for methods and factors used for blending if it is blended.

In our case, the question is: Which design criteria are more appropriate to be fed into the MCDA method? In order to better answer this question, parametric studies of design variables with respect to these design criteria are conducted first, followed by the determination of which design criteria would be further fed into MCDA method.

5.1.2 Parametric Studies of Design Criteria

The parametric study for wing thickness-to-chord ratio is illustrated in Figure 5.3. It can be observed that there are optimal settings of thickness-to-chord ratio with regard to the minimization of OEM, aircraft price, DOC, and TOM. With the increase of thickness-to-chord ratio, fuel mass, and fuel cost increase significantly. Thickness-to-chord ratio has no influence on utilization/(block time) and passenger density.

Parametric studies for aspect ratio, reference area, cruise Mach number, and fuselage diameter are presented in Figure C.1, Figure C.2, Figure C.3, and Figure C.4 and are attached in Appendix C.1, respectively.

It can be seen from Figure C.1 that there is one optimum of aspect ratio regarding the minimization of DOC. Besides, OEM, aircraft price, and TOM increase with aspect ratio, while fuel mass and fuel cost decrease. Aspect ratio has no influence on utilization/(block time) and passenger density.

It can be obtained from Figure C.2 that there are optimum points for reference area to minimize DOC and TOM. OEM and aircraft price increase with reference area, while fuel mass and fuel cost decrease. Reference area has no impact on utilization/(block time) and passenger density.

It can be seen from Figure C.3 that there are optimal points for cruise Mach number for the minimization of OEM, fuel mass, aircraft price, and TOM, respectively. Utilization/(block time), DOC, and fuel cost increase with cruise Mach number. Cruise Mach number has no influence on passenger density. It is also important to point out that there does exist optimal cruise Mach number regarding the minimization of total DOC (Euro), rather than DOC per block hour.

It is shown in Figure C.4 that OEM, fuel mass, DOC, aircraft price, fuel cost, and TOM all increase with fuselage diameter, while passenger density decreases. Fuselage diameter has no influence on utilization/(block time).

Another observation obtained from parametric studies is that all design variables un-

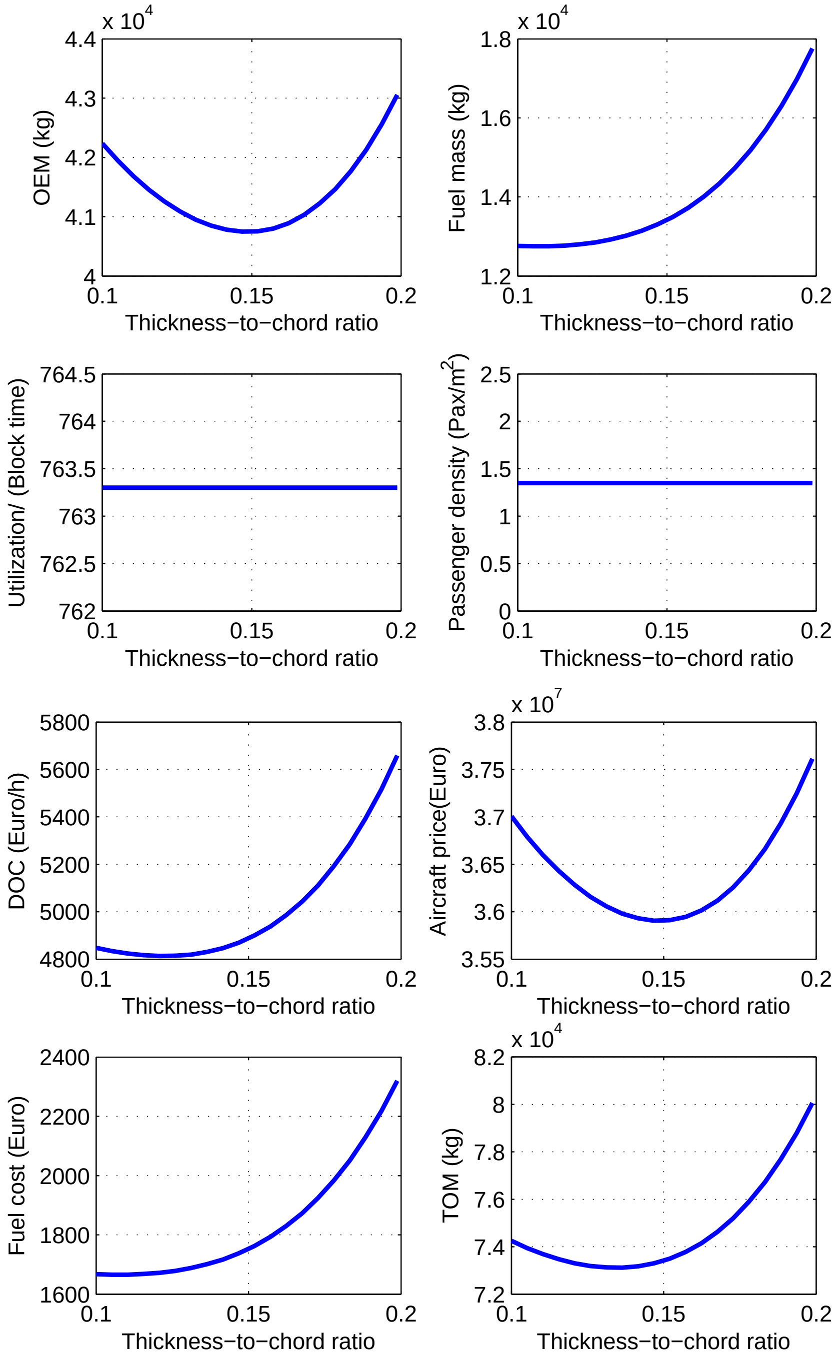


Figure 5.3: Parametric Study of Thickness-to-chord Ratio versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

der investigation are continuous, and design criteria with respect to the design variables in the conceptual aircraft design tool (VAMPzero) are rather smooth. This observation can help to choose the optimization routine for the proposed framework in Section 5.3.

Determination of Evaluation Criteria

The common practice of using DOC as objective function in the optimization is not appropriate in this study, considering that DOC has high correlation with all other design criteria. Besides, aircraft price is highly correlated to OEM, and fuel cost is calculated by fuel mass and block time. Payload is fixed (150 passenger + 5000 kg cargo) in this case, TOM is merely determined by OEM and fuel mass.

Therefore, in order to explore the interrelationships among the interest of manufacturers, the concern of fuel-based emissions, the concerns of airliners, and the consideration of passenger comfort explicitly, four design criteria: OEM, fuel mass, utilization/(block time), and passenger density, are selected to feed into the MCDA method. The other unselected design criteria: DOC, aircraft price, fuel cost, and TOM, will be traced as aircraft performance measures during the optimization.

The five design variables were listed in Table 5.1. The constraints imposed in the aircraft design process are wing span, fuel tank volume, take-off field length, landing field length, take-off wing loading, and cruise thrust. The design variables, constraints, and design criteria for this simplistic aircraft design model are summarized in Table 5.2.

Table 5.2: Summary of Design Variables, Constraints, and Design Criteria in Aircraft Optimization Process

	Units	Values
Design Variables		
Wing thickness-to-chord ratio	–	$[0.1, 0.2]$
Wing aspect ratio	–	$[8, 12]$
Wing reference area	m^2	$[80, 140]$
Cruise Mach number	–	$[0.70, 0.84]$
Fuselage diameter	m	$[3.8, 4.2]$
Constraints		
Wing span	m	≤ 36
Fuel mass	kg	$\leq$ Fuel tank volume
Take-off field length	m	≤ 3000
Landing field length	m	≤ 2000
Take-off wing loading	kg/m^2	≤ 600
Cruise thrust	N	≤ 0.9 Take-off thrust
Design Criteria		
OEM	kg	–
Fuel mass	kg	–
Utilization/(block time)	–	–
Passenger density	Pax/m^2	–

5.2 Selection of an Appropriate MCDA Method

In this section, the selection of the most appropriate MCDA method for the aircraft design problem is presented, through the developed intelligent multi-criteria decision support system, as described in Chapter 3. The user guide can be found in Appendix B. The step by step method selection process is explained and discussed in the following subsections.

Step 1: Define the Problem

As discussed in Section 5.1, the decision making problem in this simplistic aircraft design is to aggregate the four design criteria into one compound figure of merit using one appropriate MCDA method. The proposed intelligent multi-criteria decision support tool will be employed to facilitate this decision making process.

Step 2: Define the Evaluation Criteria

In order to identify the most appropriate method, sixteen widely used MCDA methods are studied and their characteristics are stored in a method database. To compare the appropriateness of the methods with respect to the given problem, each method is evaluated based on the proposed twelve evaluation criteria. The twelve evaluation criteria can be captured by answering twelve questions relevant to the characteristics of the methods, as shown in Figure 5.4.

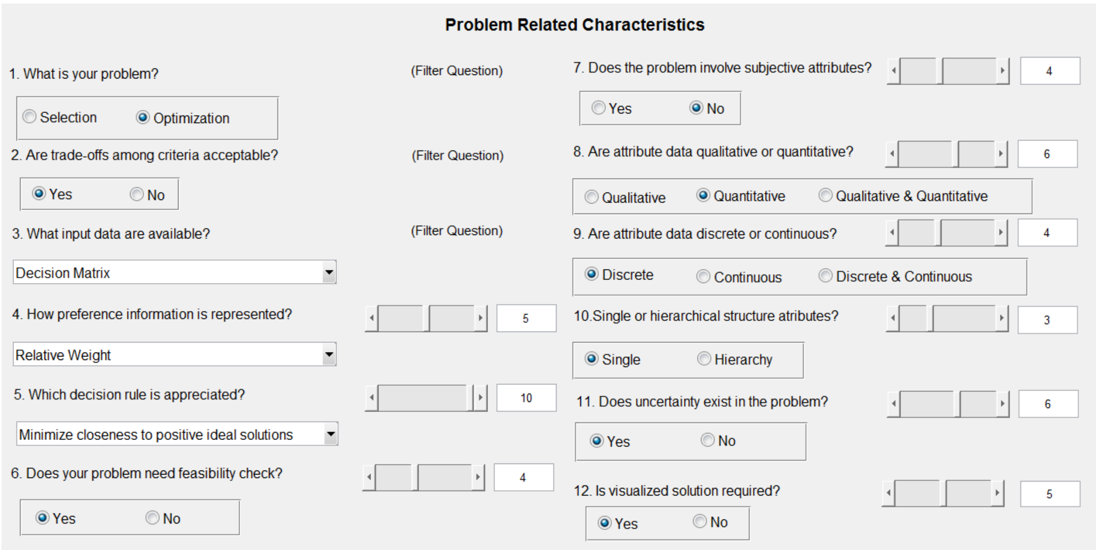


Figure 5.4: Questions Related to Evaluation Criteria for Method Selection in Aircraft Design Process

Step 3: Perform Initial Screening

In this step, infeasible MCDA methods are eliminated first by three filter questions. Considering that in this aircraft design problem, the compound figure of merit for the four design criteria aggregated by MCDA method serves as objective function in the optimization, scoring methods are more appropriate than classification methods. Meanwhile, all non-compensatory methods are excluded since compensation is allowed in the aircraft optimization process.

Step 4: Define the Preferences on Evaluation Criteria

Since the DM may consider one criterion is more important than the other when selecting the most appropriate method, relative weight is defined for each criterion to reflect the DM's preference information. The DM's preference information on the evaluation criteria can be defined using slide bars in the integrated user interface, with a subjective scale of 0 to 10, where 0 stands for extremely unimportant while 10 represents extremely important.

Step 5: Calculate the Appropriateness Index

In essential, Appropriateness Index (AI) is used to determine how the characteristics of a method match the characteristics of the given decision making problem. In this step, AI for each MCDA method is calculated by using Equation 3.1, as described in Subsection 3.2.5 in Chapter 3.

Step 6: Evaluate the MCDA methods

Based on the calculation, AI of the MCDA methods are obtained and shown in Figure 5.5, where higher score represents more appropriateness of the method when solving the given problem.

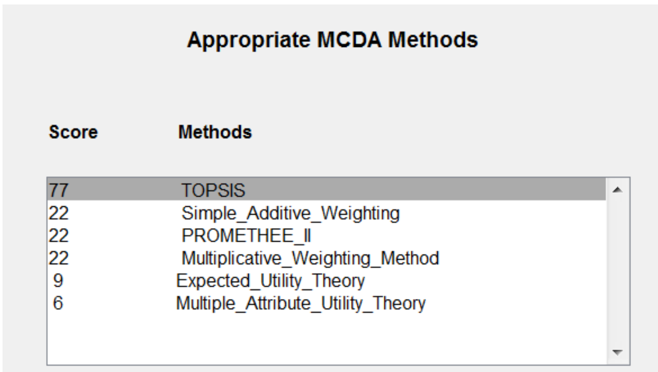


Figure 5.5: MCDA Methods Ranking List with Scores in Aircraft Design Process

Step 7: Choose the Most Suitable Method

In this example, as indicated in Figure 5.5, TOPSIS gets the highest score among the MCDA methods, therefore, it is selected as the most appropriate method to solve the aircraft design problem. In our user friendly interface, the DM can simply click the name of the method and methodology instructions of TOPSIS will be displayed to guide the DM to solve the given problem, as illustrated in Figure 5.6.

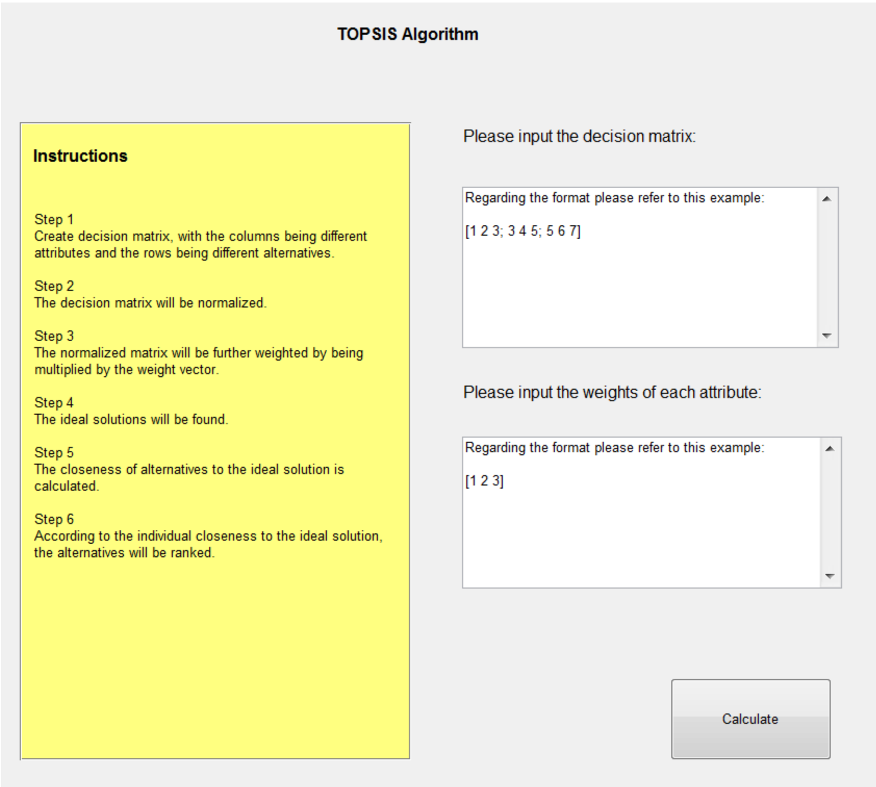


Figure 5.6: Methodology Instructions for TOPSIS

An Improved TOPSIS (ITOPSIS)

In the original TOPSIS method, when an alternative is removed from or added to the candidate alternatives, the two hypothetical ideal solutions will probably change and the Euclidean distances to the two hypothetical ideal solutions will also change. Thus, the top-ranked alternative would possibly become inconsistent when the candidate alternatives are changed. It has been pointed out that the cause of rank inconsistency with TOPSIS lies in the calculation step of determining the two hypothetical ideal solutions (19). A pair of absolute ideal solutions instead of the relative ideal solutions was introduced to eliminate the rank inconsistency of TOPSIS method.

In this study, an Improved TOPSIS (ITOPSIS) will be utilized to aggregate the four design criteria into one compound figure of merit for optimization. The positive ideal solution and negative ideal solution are set beforehand in order to avoid the ranking inconsistency. In our case, two kinds of optimizations are conducted for each of the four design criteria: minimization and maximization. The positive ideal solutions and negative ideal solutions for the four design criteria are searched within design space of eight optimizations, as summarized in Table 5.3. It should be noted that the utilization/(block time) ratio is a benefit criterion, and the other three are cost criteria.

Table 5.3: The Positive Ideal Solution and Negative Ideal Solution in ITOPSIS

Ideal solutions	OEM (kg)	Fuel mass (kg)	Utilization/ (block time)	Passenger density (Pax/m^2)
Positive	36943.4992	11766.8787	796.8551	1.2875
Negative	50521.0972	20864.0399	715.0679	1.4063

Step 8: Conduct Sensitivity Analysis

Since different DMs often have different preference information on the evaluation criteria and different answers to the twelve questions, sensitivity analysis to the variation of DM's preference information and the input data should be performed on the MCDA method selection process. In our integrated user friendly interface, the DM can adjust the weights of each criterion by moving the corresponding slide bars. If the DM is satisfied with the final results, the solution can be implemented. Otherwise, the DM can go back to Step 2 and modify the input data or preference information and repeat the selection process until a satisfying outcome is obtained.

In this example, with the current preference information and input data, it can be seen from Figure 5.5 that SAW, PROMETHEE, and multiplicative weighting method, are ranked second by the multi-criteria decision support system. According to the methodology description in Chapter 2, PROMETHEE needs three threshold values for each criterion: indifference threshold, strict preference threshold, and an intermediate value between indifference and strict preference threshold. These extra twelve thresholds for the four design criteria increases the complexity of the aircraft design problem significantly. Moreover, these extra twelve threshold values are rather subjective and different DMs often have different threshold values. Besides, the difference between SAW and multiplicative weighting method is the multiplicative property of the weighting factors. Therefore, considering that SAW is one widely used MCDA method, it will be used in the aircraft design problem for the purpose of comparison.

5.3 Proposed Multi-Criteria Optimization Framework

Considerable research has been devoted to the development of optimization methods in order to deal with multiple, conflicting objectives (criteria), such as multi-objective Genetic Algorithms (GA) (23). For instance, a three-objectives GA was used to explore the trade-offs between noise, emissions, and operating costs in the aircraft conceptual design stage (5). A two-objectives GA was applied to balance fuel, NO_X emission, and DOC (47). However, multi-objective GA suffer from expensive computation. Furthermore, evolutionary multi-objective optimization techniques are not easily applicable for handling a large number of objectives (24).

A new multi-criteria optimization framework incorporating MCDA techniques in aircraft conceptual design process is established, as illustrated in Figure 5.1. ITOPSIS is utilized to aggregate the multiple design criteria into one composite figure of merit. This composite figure of merit serves as an objective function during the optimization. This framework supports the designer to quickly assess the compromised design alternatives. Moreover, the MCDA techniques have the ability to handle large number of objectives.

In this section, optimization algorithms are briefly reviewed first. Then, optimization results of typical weighting scenarios are presented. At last, optimization using ITOPSIS index and SAW index as objective functions are compared.

5.3.1 Numerical Optimization Techniques

There are several optimization algorithms currently available, among which gradient-based methods and genetic algorithms are most widely used in aircraft design.

Gradient-based methods compute the gradient of the objective function with respect to design variables, the gradient vector establishes a search direction of the deepest slope, the objective function changes most rapidly in this direction (45). Gradient-based methods can provide efficient design solutions. However, Gradient-based methods have problems with discontinuous functions and functions that have discrete variables. In addition, when the objective function varies in a non-smooth fashion, gradient-based methods may have the risk of ending up in a local optimum.

GA are stochastic evolutionary algorithms inspired by biological evolution, they operate on a population of candidate solutions and apply the principle of survival of the fittest to evolve the candidate solutions towards the desired optimal solutions (23). Continuous and discrete variables can be included in GA simultaneously, where the continuous variables are discretized with a reasonable resolution. Additionally, GA consider the whole design space, thus, the risk of convergence to a local optimum can be avoided. However, GA suffer from expensive computation, and different optimization runs may result in different optimal solutions.

Which optimization method to use depends on the optimization problem under consideration. If all desig variables are continuous and objective functions are smooth, gradient-based methods should be used in the optimization process. If there are discrete variables and objective functions are noisy, GA should be employed.

According to parametric studies performed in the previous Subsection 5.1.2, it is observed that all design variables under investigation are continuous, and objective functions with respect to the design variables in the conceptual aircraft design tool (VAMPzero) are rather smooth. Therefore, gradient-based methods are used in the established optimization framework.

Evaluation of Gradient-based Optimization with Different Starting Points

It is important to note that gradient-based methods are prone to finding a local optimum, depending on the location of the starting point. In order to assess whether the gradient-based optimizer (sequential quadratic programming algorithm) can converge towards the same optimal design in the aircraft optimization process, optimization tests using ITOPSIS index as an objective function starting from different initial points are conducted in this subsection.

The baseline and ranges for the five design variables under consideration were summarized in Table 5.1 in Section 5.1. Random starting points are generated within their lower bounds and upper bounds, as shown in Equation 5.6

$$(\text{upper bound} - \text{lower bound}) \times \text{random number} + \text{lower bound} \tag{5.6}$$

where $0 \leq$ random number ≤ 1. The lower bounds and upper bounds of design variables are the minimum values and maximum values scaled against baseline. Ten sets of random starting points are listed in Table 5.4. The optimized design using these ten sets of different starting points are summarized in Table 5.5.

Table 5.4: Ten Sets of Random Starting Points in the Optimization Process

Set	Thickness-to-chord ratio	Aspect ratio	Reference area (m^2)	Cruise Mach number	Fuselage diameter (m)	Optimization time (s)
1	0.1058	10.0875	96.5858	0.7763	4.1628	1165
2	0.1995	8.6434	137.8836	0.8358	4.1288	606
3	0.1310	9.5031	81.0778	0.7455	4.0204	3666
4	0.1406	11.4378	115.9128	0.7674	4.1024	400
5	0.1151	9.1611	127.7489	0.7323	4.0620	390
6	0.1551	11.1465	132.9876	0.7114	3.8276	382
7	0.1266	10.4230	88.1035	0.8208	3.9396	442
8	0.1610	11.6172	105.6067	0.8032	3.8596	392
9	0.1763	8.1698	100.1110	0.7870	3.8872	483
10	0.1889	9.9757	119.2910	0.7268	3.9680	339

Table 5.5: The Optimized Design using Ten Sets of Random Starting Points

Set	Thickness-to-chord ratio	Aspect ratio	Reference area (m^2)	Cruise Mach number	Fuselage diameter (m)
1	0.1349	9.3783	116.9663	0.7603	3.8
2	0.1344	9.3697	116.9928	0.7611	3.8
3	0.1350	9.3923	116.9975	0.7613	3.8
4	0.1351	9.3999	116.9855	0.7600	3.8
5	0.1349	9.3929	116.9864	0.7601	3.8
6	0.1347	9.3733	116.9708	0.7606	3.8
7	0.1351	9.4015	116.9810	0.7596	3.8
8	0.1351	9.4014	116.9878	0.7599	3.8
9	0.1349	9.3948	116.9891	0.7600	3.8
10	0.1350	9.3954	116.9825	0.7600	3.8

It is observed that the gradient-based optimizer is able to find the same optimal design starting from different initial points. Furthermore, computation times of the optimization starting from different initial points have also been recorded. It is noted that the Set 1 and Set 3 took unusual longer time than other sets, this can be attributed to the fact that the starting points of reference area and thickness-to-chord ratio are far away from the optimal design, thus, the optimizer needs more iterations to converge towards the optimal design solution.

5.3.2 Optimization Results of Typical Weighting Scenarios

In this section, several typical weighting scenarios in the optimization process are investigated, ranging from one criterion preferred to evenly distributed. This is one approach to simulate DM's preference information. Optimization results for single criterion are summarized in Table 5.6, and optimization results with equal weighting factors among the four design criteria are summarized in Table 5.7, respectively.

Table 5.6: Optimization Results for Single Criterion

	Baseline Design	Min OEM	Min Fuel mass	Max Utilization/ (block time)	Min Passenger density
Design Variables					
Thickness-to-chord ratio	0.13	0.1585	0.1220	0.1286	0.1301
Aspect ratio	9.4	8.0347	11.6740	9.3237	9.3608
Reference area (m^2)	122.40	116.18	132.05	128.53	125.77
Cruise Mach number	0.78	0.71	0.73	0.84	0.77
Fuselage diameter (m)	4	3.8	3.8	3.9	4.2
Design Criteria					
OEM (kg)	40980	36949	43725	42974	42426
Fuel mass (kg)	12903	13280	11771	15319	13312
Utilization/(block time)	763	722	734	797	759
Passenger density (pax/m^2)	1.35	1.4211	1.4211	1.3863	1.2981
Traced Performance Measures					
DOC (Euro/h)	4818	4577	4672	5402	4925
Aircraft price (Euro)	36077718	33100305	38106043	37551218	37146224
Fuel cost (Euro/h)	1685	1626	1470	2104	1728
TOM (kg)	73133	69479	74746	77544	74988

Table 5.7: Optimization Results when Weighting Factors are Evenly Distributed

	Baseline Design	Optimized Design	Relative Change (%)
Design Variables			
Thickness-to-chord ratio	0.13	0.135	3.84
Aspect ratio	9.396	9.414	0.19
Reference area (m^2)	122.4	117.01	-4.40
Cruise Mach number	0.78	0.76	-2.55
Fuselage diameter (m)	4	3.8	-5
Design Criteria			
OEM (kg)	40980	38705	-5.55
Fuel mass (kg)	12903	12242	-5.12
Utilization/(block time)	763	752	-1.53
Passenger density (pax/m^2)	1.35	1.4211	5.26
Traced Performance Measures			
DOC (Euro/h)	4818	4588	-4.76
Aircraft price (Euro)	36077718	34397326	-4.66
Fuel cost (Euro/h)	1686	1571	-6.79
TOM (kg)	73133	70197	-4.01

It can be seen from Table 5.6, when optimizing OEM, fuselage diameter is reduced to the lower boundary, aspect ratio is reduced by 14% based on baseline design (close to the lower limit 8), reference area is decreased by 5%, and thickness-to-chord ratio is increased by 21%. The decrease of aspect ratio and reference area leads to a reduction in wing weight, which contributes to a reduction in OEM and TOM. As expected, aircraft price is also reduced by 8% because of the reduction in OEM. Fuel cost is reduced by 4% and DOC is decreased by 5%. However, the decrease of aspect ratio and reference area and the increase of thickness-to-chord ratio result in an increment of the overall drag of the aircraft and 9% reduction in cruise Mach number. The reduction in cruise Mach number leads to a 5% decrease in utilization/(block time). It also requires more fuel to fly the mission range. Besides, the decrease of fuselage diameter leads to a 5% increase of passenger density.

When optimizing the aircraft for fuel mass, aspect ratio is increased by 24%, reference area is increased by 8%, and thickness-to-chord ratio is decreased by 6%. The increase of aspect ratio and reference area leads to a larger span and an increase in wing weight, which further leads to the increase of OEM, TOM, and aircraft price. Flying slower (low cruise Mach number) can also reduce the consumption of fuel for certain mission range. However, lower cruise Mach number will prolong block time, thus, utilization/(block time) ratio is decreased. In addition, the overall drag of the aircraft can be reduced when the wetted area of fuselage is reduced, this is the reason why fuselage diameter is decreased to the lower boundary.

When optimizing the aircraft for utilization/(block time) ratio, cruise Mach number is increased to the upper boundary, fuselage diameter is reduced so that the wet area of fuselage is reduced, reference area is increased by 5%. The decrease of fuselage diameter and increase of reference area lead to the reduction of the overall drag of the aircraft. However, the increase of cruise Mach number will burn more fuel for specific mission range, thus, fuel mass and fuel cost are increased 19% and 25%, respectively. DOC is also increased by 12%, considering the dominant role of fuel cost. The increase of reference area leads to the increase of OEM, TOM, and aircraft price. Additionally, the decrease of fuselage diameter results in 4% increase of passenger density.

When optimizing the aircraft for passenger density, fuselage diameter is increased to its upper limit. Reference area is increased slightly by 3%, thickness-to-chord ratio, aspect ratio, and cruise Mach number almost do not change. Except utilization/(block time) ratio has decreased slightly, all other criteria have been increased by around 2.5%.

The conflicting design criteria are further explored when weighting factors are evenly distributed, as summarized in Table 5.7. Thickness-to-chord ratio is increased by 4%, aspect ratio almost does not change, reference area is decreased by 4%, cruise Mach number is decreased by 2.5%, and fuselage diameter is decreased to its lower boundary. The reduction of OEM and fuel mass is compromised by the decrease of utilization/(block time) ratio and the increase of passenger density.

Moreover, it can be observed from Table 5.7 that except for utilization/(block time) ratio is decreased by 1.5%, the other three design criteria have around 5% change. Therefore, utilization/(block time) ratio is less sensitive than other three design criteria in the aircraft design process.

The similar observation can be obtained when the relative changes of the four traced aircraft performances are compared. Fuel cost is decreased by around 6%, while the other three traced aircraft performances are all decreased by around 4%. Thus, fuel cost is more sensitive than other three traced aircraft performances in the aircraft design process.

5.3.3 Comparison Using Different MCDA Indices as Objective Functions

For the purpose of comparison, the proposed optimization framework is also performed when using SAW index as an objective function, optimization results are summarized in Table 5.8. The comparison of relative changes for the design criteria and traced performance measures, when using ITOPSIS index as an objective function (Table 5.7) and SAW index as an objective function (Table 5.8), are presented in Figure 5.7.

Table 5.8: Optimization Results using SAW Index as an Objective Function, when Weighting Factors are Evenly Distributed

	Baseline Design	Optimized Design	Relative Change (%)
Design Variables			
Thickness-to-chord ratio	0.13	0.1304	0.28
Aspect ratio	9.396	9.118	-2.95
Reference area (m^2)	122.4	116.9	-4.48
Cruise Mach number	0.78	0.77	-1.50
Fuselage diameter (m)	4	3.8	-5
Design Criteria			
OEM (kg)	40980	38552	-5.92
Fuel mass (kg)	12903	12344	-4.33
Utilization/(Block time)	763.3	756.5	-0.89
Passenger density (pax/m^2)	1.35	1.4211	5.26
Traced Performance Measures			
DOC (Euro/h)	4818	4612	-4.27
Aircraft price (Euro)	36077718	34284714	-4.97
Fuel cost (Euro/h)	1686	1596	-5.32
TOM (kg)	73133	70147	-4.08

It is observed from Figure 5.7 that with equally assigned weighting factors, the optimized design using ITOPSIS index as an objective function is heavier but more fuel efficient than the design which is optimized using SAW index as an objective function.

Furthermore, in the same running environment (Windows 7, 2.66 GHz Intel Core 2 Quad CPU, 4 GB RAM, and Matlab 2010a version), convergence rates when using ITOPSIS index and using SAW index as objective functions are summarized in Table 5.9. It is seen that the optimization using ITOPSIS index as an objective function needs less iterations and less computation time than using SAW index as an objective function.

Table 5.9: Comparison of Convergence Rates, using ITOPSIS Index and SAW Index as Objective Functions

Objective function	Iterations	Optimization time (seconds)
ITOPSIS index	5	304
SAW index	39	3005

However, only with the conduction of one set of weighting factors, it cannot be concluded that which MCDA method is more appropriate for the optimization, considering the crucial impact of weighting factors on the optimized design. The roles of weighting factors in the framework of incorporating MCDA techniques in aircraft design will be further investigated in the following section.

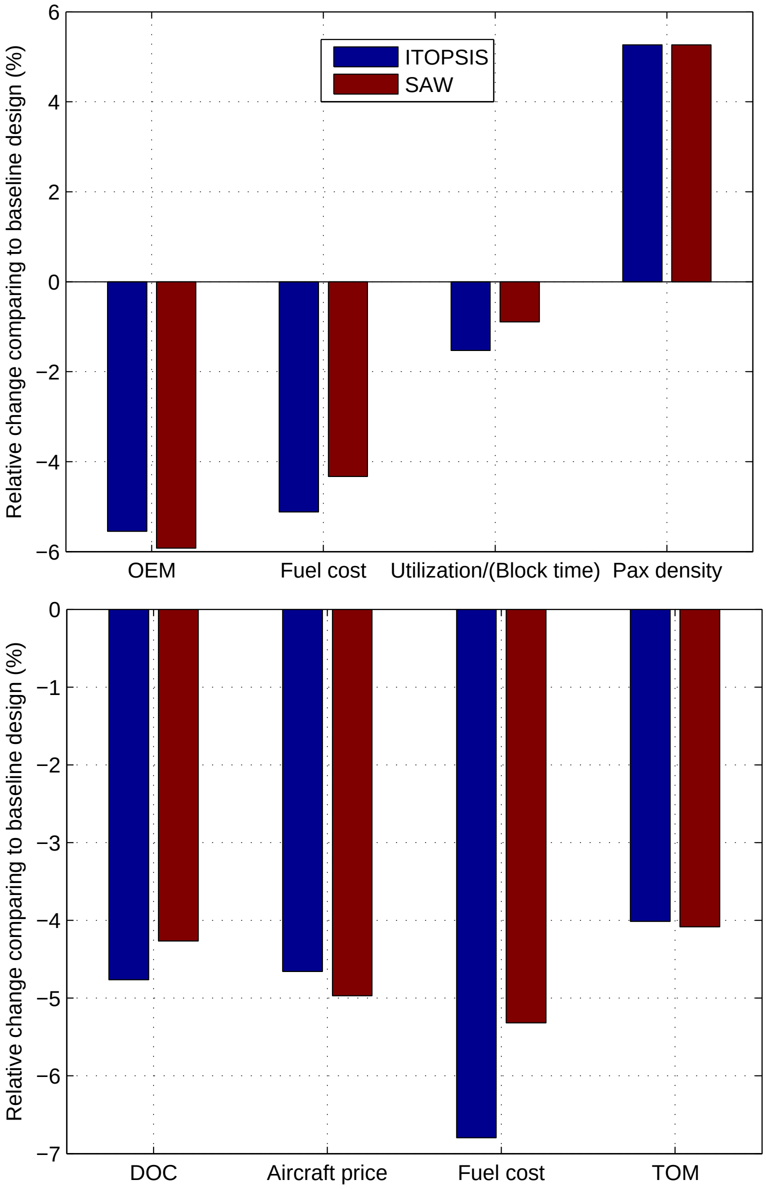


Figure 5.7: Comparison of Relative Changes for Design Criteria and Traced Performance Measures, using ITOPSIS Index and SAW Index as Objective Functions

5.4 Surrogate Model Construction for Design Criteria in terms of Weighting Factors

Weighting factors create a compound figure of merit. The compound figure of merit serves as the objective function for optimization. Different weighting schemes result in different compound figure of merits. The selection of weighting factors is critical to the determination of an optimal design, since if a design is optimized to the wrong figure of merit, it will not be the best design in terms of the real important measure.

Especially, inherent uncertainties and subjectivities of the weighting factors have significant impacts on the design solution. An uncertainty assessment that demonstrates this impact must consider different combinations of weighting factors. However, in the proposed multi-criteria optimization framework, the computation time for one set of weighting factors is at least 5 minutes. A Monte Carlo based uncertainty analysis with 10,000 samples would take at least 35 days. The long computation time makes the uncertainty assessment an intractable computational task.

In this study, surrogate models for the four design criteria in terms of weighting factors are constructed. Each point of this surrogate model represents an optimized aircraft design for a given set of weighting factors. The whole framework of incorporating MCDA techniques in aircraft design process is treated as a black box. An overview of surrogate modeling process for design criteria in terms of weighting factors is shown in Figure 5.8. The constructed surrogate model provide efficient analysis tools for uncertainty assessment.

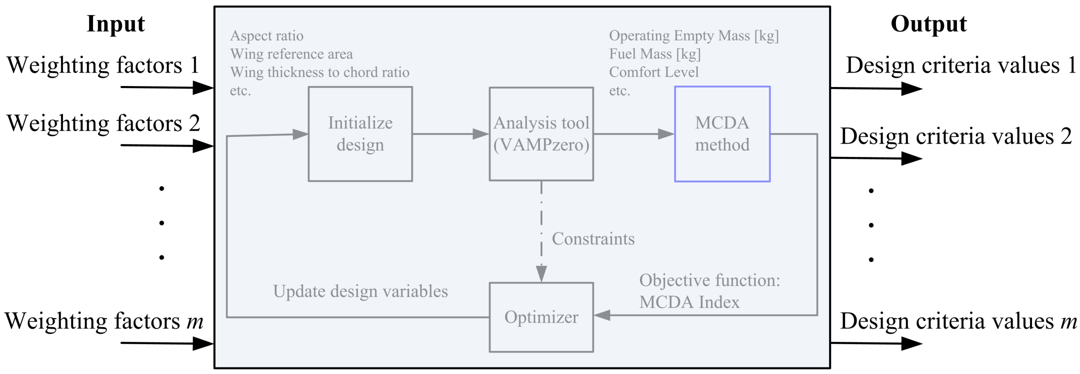


Figure 5.8: Overview of Surrogate Modeling Process for Design Criteria in terms of Weighting Factors

There are typically four steps in surrogate model building process: sample the design space using experimental design, choose a model to represent the input and output data, select a method to fit the model, and validate the constructed model (30). The construction of surrogate models for design criteria in terms of weighting factors will follow this process. Each step is discussed in detail in the following subsections.

5.4.1 Experimental Design

Experimental design is a sequence of experiments to be performed, expressed in terms of factors set at specified levels (63). Experimental designs were originally developed for effective physical experiments, they are being applied to computer experiments with the purpose of reducing the computation time and increasing the efficiency.

In order to explore the design space thoroughly, experimental design with spatially uniform distribution is one effective approach. There are several space filling strategies (50), among which Latin Hypercube Sampling (LHS) is one reliable method to generate random candidate samples, with guarantee that these samples are relatively uniformly distributed in the design space (55).

In this study, weighting factors $W = (w_1, w_2, ..., w_n)$ generated by experimental design have to satisfy two conditions:

1. $0 \leq w_i \leq 1$
2. $\sum_{i=1}^{n} w_i = 1$

When standard LHS is utilized to generate m sets of weighting factors for n criteria ($W_{m\times n}$), for each experimental run, the factor setting W_{ij} $(i = 1, 2, ..., m, j = 1, 2, ..., n)$ is randomly sampled from each interval $(0, 1/m), (1/m, 2/m), ..., (1 - 1/m, 1)$. The standard LHS meets the condition 1 that all the factor settings range from 0 to 1. However, for each experimental run, the sum of the factor settings in each run does not equal to 1. The normalization of the factor settings can fulfill the condition 2, however, the hypercube will be deformed and the Latin properties may not be guaranteed.

In this case, in order to generate experimental designs fulfilling the two conditions, standard LHS is conducted first, then the samples generated by LHS are rectified by Dirichlet distribution.

One Modified LHS with Dirichlet Distribution

Dirichlet distribution is a family of continuous multivariate probability distributions parameterized by a vector $\alpha = (\alpha_1, \alpha_2, ..., \alpha_k,)$ of positive reals. Dirichlet distribution is one multivariate generalization of beta distribution and is defined as Equation 5.7

$$Dir(X, \alpha) = \frac{\Gamma(\alpha_1 + \alpha_2 + ... + \alpha_k)}{\Gamma(\alpha_1)\Gamma(\alpha_2)...\Gamma(\alpha_k)} \prod ({x_1}^{\alpha_1 - 1} {x_2}^{\alpha_2 - 1} ... {x_k}^{\alpha_k - 1}) \tag{5.7}$$

where $X = (x_1, x_2, ..., x_{k-1})$, satisfying $x_i > 0$ and $\sum_{i=1}^{k-1} x_i < 1$. Besides, $x_k = 1 - x_1 - x_2 - ... - x_{k-1}$. Symmetric Dirichlet distribution is when the components of vector α are equal. If each component of α is 1, the symmetric Dirichlet distribution is equivalent to a uniform distribution; if each component of α is bigger than 1, it prefers dense, evenly distributed distribution, and if each component of α is smaller than 1, it prefers sparse distribution.

When using the modified LHS with Dirichlet distribution, although the modified sample values are not strictly uniformly distributed any more, Dirichlet distribution can keep the ranges of the sample values larger once they are normalized, while maintaining the appealing Latin properties.

One Example of Standard LHS, Normalized LHS, and Modified LHS with Dirichlet Distribution

One example of standard LHS, normalized LHS, and the modified LHS with Dirichlet distribution is demonstrated as follows. In order to generate ten sets of weighting factors for three criteria, standard LHS is conducted firstly, as shown in Figure 5.9, where S_1, S_2, and S_3 represent the sample values for the three criteria. It is noted that there is exactly one point in each row and each column in the two dimensional projections, and the sample values range from 0 to 1 (which meets the condition 1), however, the sum of one set of the sample values is not equal to 1 (which does not meet the condition 2).

Thus, in order to fulfill condition 2, standard LHS can be normalized by its row sum, as shown in Figure 5.10, where Lw_1, Lw_2, and Lw_3 represent the normalized sample values for the three criteria. It is observed that the range of the normalized sample values shrinks into 0 to 0.8. Moreover, there is no point in the bins which are bigger

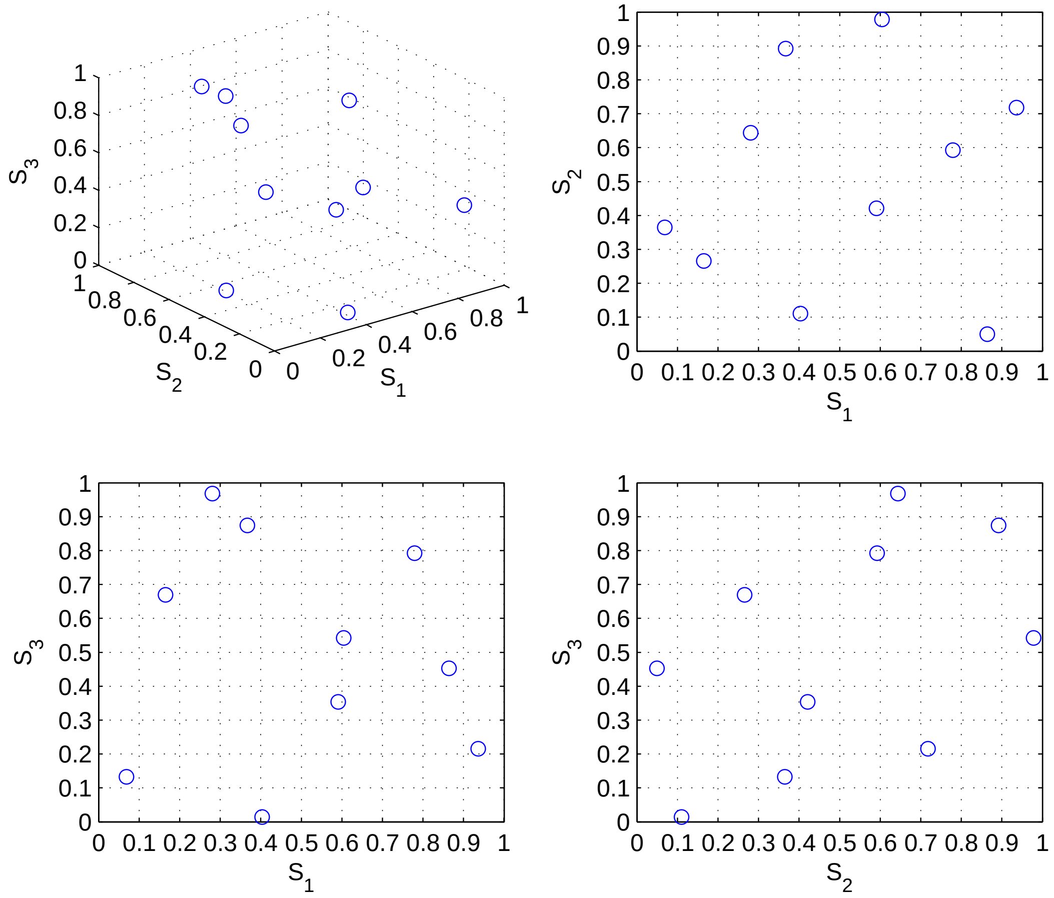


Figure 5.9: Standard Latin Hypercube Sampling in Three Dimensions and with Two Dimensional Projections

than 0.8, thus, the hypercube is deformed and the Latin properties is not maintained.

The modified LHS with Dirichlet distribution are shown in Figure 5.11, where LDw_1, LDw_2, and LDw_3 represent the sample values rectified by Dirichlet distribution for the three criteria. It is observed that the range of the sample values are recovered from 0 to 1, although there is not exactly one point in each row and each column in the two dimensional projections.

In this study, one hundred sets of weighting factors are generated by the modified LHS with Dirichlet distribution. The data is attached in Table D.1 in Appendix D.1. The weighing factors reflect the relative importance of the design criteria. For instance,

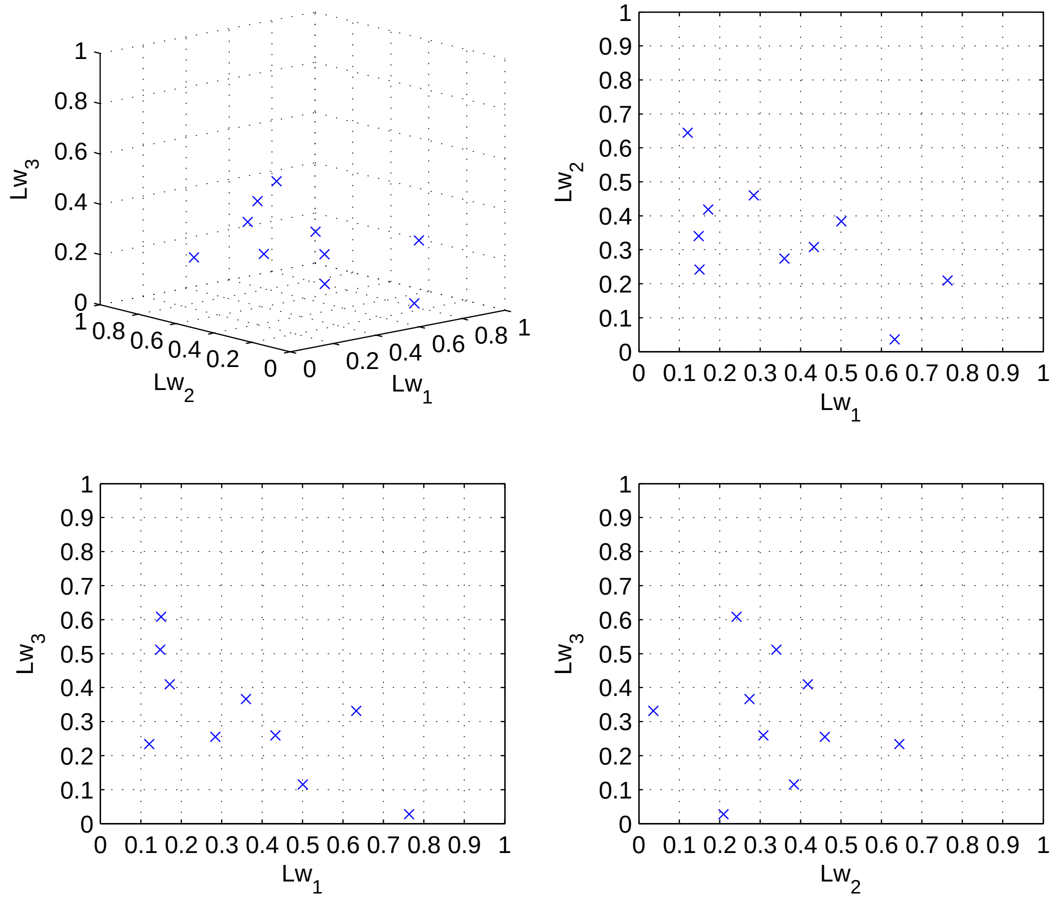


Figure 5.10: Normalized Latin Hypercube Sampling by Its Row Sum in Three Dimensions and with Two Dimensional Projections

the first row in Table D.1 is $W_1 = [0.4333\ 0.0176\ 0.3719\ 0.1772]$. This set of weighting factors indicates that the first design criterion (OEM) is most important, followed by the third design criterion (utilization/(block time) ratio) and the fourth design criterion (passenger density), while the second design criterion (fuel mass) is least important. The other 99 sets of weighting factors have similar explanations.

5.4.2 Model Choice

Response surface is one popular approach to build surrogate models (63). Response surface typically involves least square regression to fit a polynomial model of the observed response values. The most common response surface models are low-order polynomials.

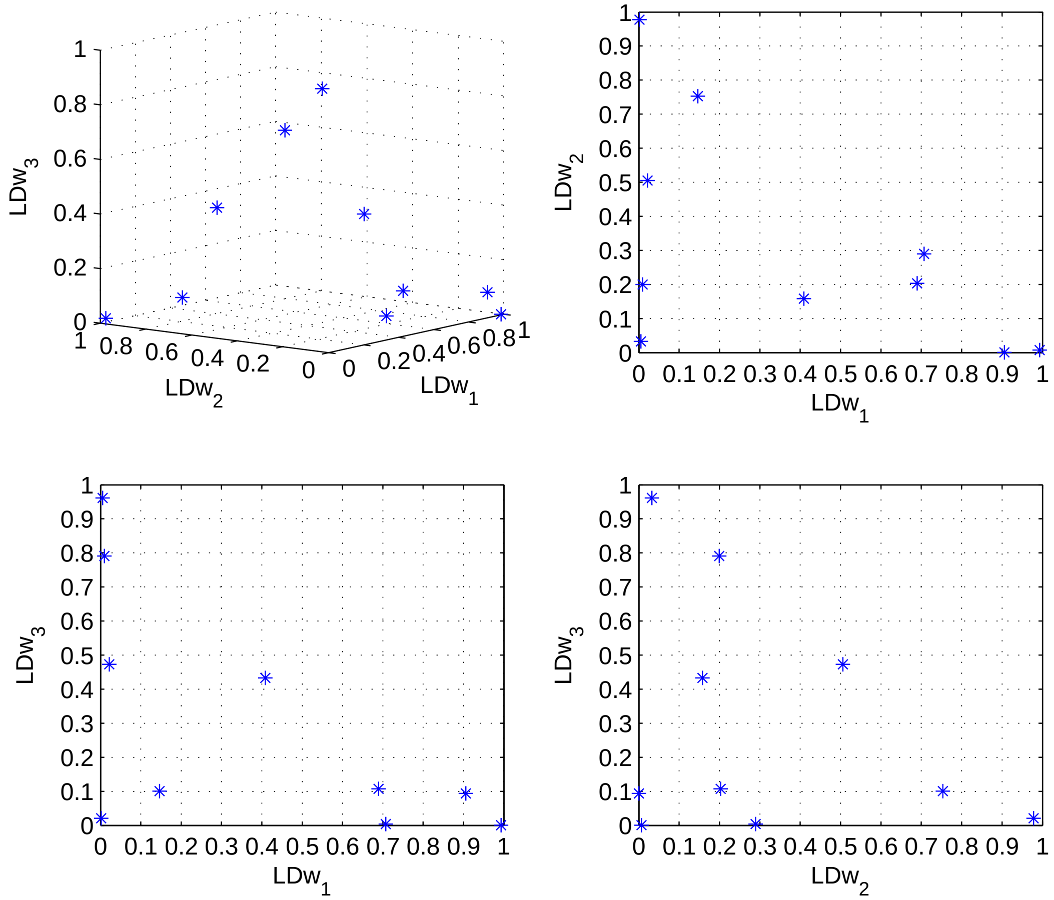


Figure 5.11: Modified Latin Hypercube Sampling with Dirichlet Distribution in Three Dimensions and with Two Dimensional Projections

For an unknown function of interest $y(x)$, as defined in Equation 5.8

$$y(x) = f(x) + \epsilon \tag{5.8}$$

where $f(x)$ is a polynomial function, ϵ is random error, which is normally distributed with mean zero and variance σ^2. A second-order polynomial model is shown in Equation 5.9. The parameters of the polynomial in Equation 5.9 are determined through least square regression, which minimizes the sum of the squares of the predicted values $\hat{y}(x)$ from the actual values $y(x)$.

$$\hat{y}(x) = a_0 + \sum_{i=1}^{k} a_i x_i + \sum_{i=1}^{k} a_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} a_{ij} x_i x_j + \epsilon \tag{5.9}$$

Response surface models have been widely used in the surrogate model construction in engineering design. There are several advantages using response surface models, such as ease of implementation, minimal efforts required to train models, and ideality for uncertainty analysis. In this research, response surface is utilized to construct the surrogate models.

5.4.3 Model Fitting

A widely used statistics software package JMP@ is employed to fit response surface models. Before the construction of response surface models, the correlations among the four design criteria and the traced aircraft performances are assessed. The pairwise correlation coefficients are summarized in Table 5.10.

Table 5.10: Pairwise Correlation Coefficients for Design Criteria of Interest

Correlations	OEM	Fuel Mass	Utilization/ (Block time)	Passenger Density	DOC	Aircraft Price	Fuel Cost	TOM
OEM	1.0000	-0.1879	0.4779	-0.4840	0.6573	1.0000	0.0781	0.9613
Fuel Mass		1.0000	0.1535	-0.5872	0.5480	-0.1879	0.8811	0.0899
Utilization/ (Block time)			1.0000	-0.1202	0.7498	0.4779	0.6013	0.5277
Passenger Density				1.0000	-0.6845	-0.4840	-0.5352	-0.6554
DOC					1.0000	0.6573	0.8026	0.8202
Aircraft Price						1.0000	0.0781	0.9613
Fuel Cost							1.0000	0.3263
TOM								1.0000

It is observed from Table 5.10 that DOC shows high correlation with all other criteria, the correlation coefficient between aircraft price and OEM is 1, fuel cost is highly correlated with fuel mass, and TOM have strong correlation with OEM. These observations are consistent with the analytical explanation of the determination of design criteria, as described in Section 5.1. Thus, Table 5.10 serves as one evidence that the selected four design criteria are more appropriate to be fed into the MCDA method for aggregation.

5.4.4 Model Validation

In this subsection, the accuracy of response surface models is assessed by the actual versus predicted plots for each response first, and is further evaluated by running additional untried data points.

Model Accuracy Evaluation by the Actual Versus Predicted Plots

The actual values versus the predicted values for the four design criteria aggregated by ITOPSIS are shown in Figure 5.12. For the purpose of comparison, the actual values versus the predicted values for the four design criteria aggregated by SAW are also conducted and are shown in Figure 5.13. In the actual by predicted plot, the horizontal dotted blue line represents the mean of actual values, the red line shows 45 degree diagonal line, and the two red dotted lines show 95% confidence intervals.

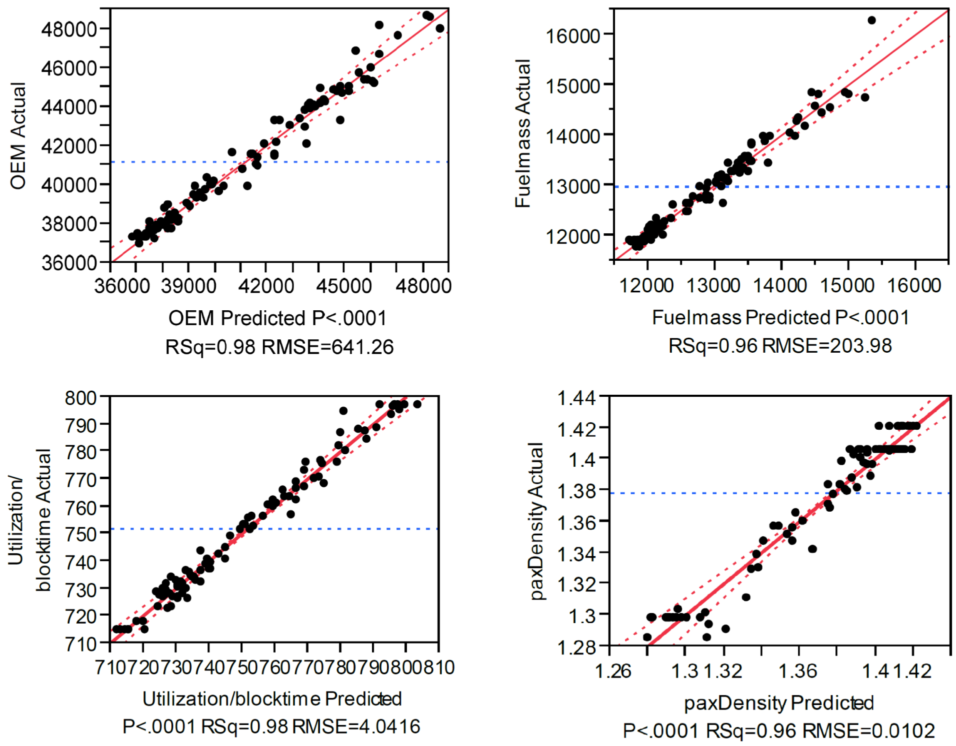


Figure 5.12: The Actual by Predicted Plots of OEM, Fuel Mass, Utilization/(Block time), and Passenger Density, when using ITOPSIS Index as an Objective Function

The actual by predicted plots illustrate how well the predicted responses match the actual data. A quick assessment of the model is to eyeball a 45 degree pattern in these plots. In our case, the scatter plots when ITOPSIS is used for the multiple criteria

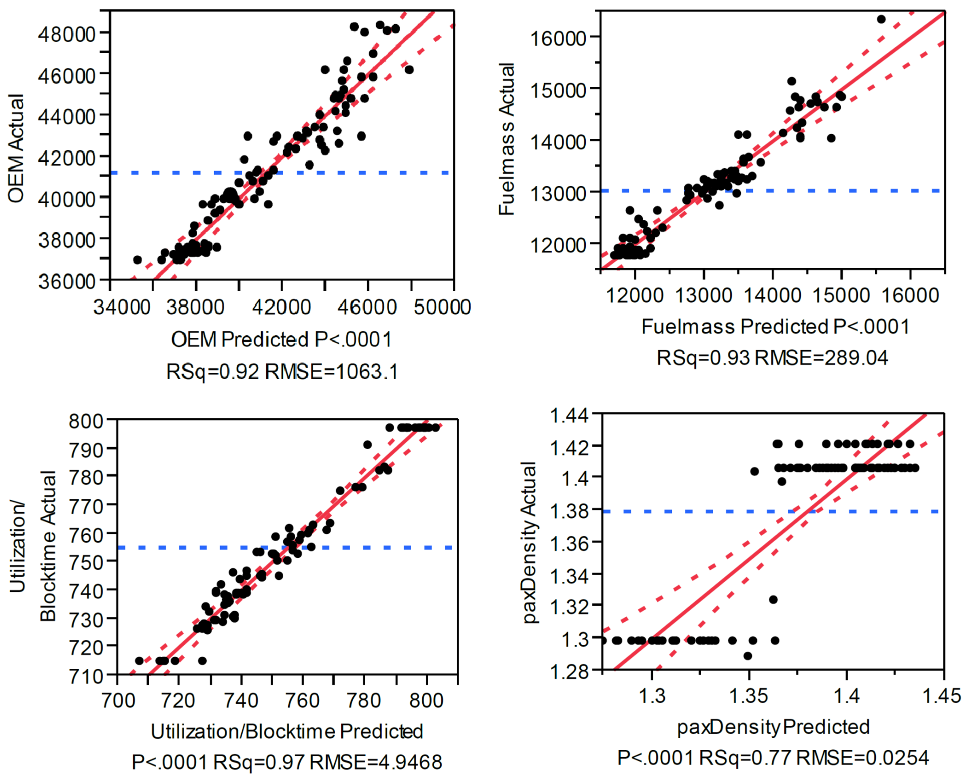


Figure 5.13: The Actual by Predicted Plots of OEM, Fuel Mass, Utilization/(Block time), and Passenger Density, when using SAW Index as an Objective Function

aggregation and when SAW is used for the multiple criteria aggregation all follow a 45 degree pattern. Specifically, the scatter plots for ITOPSIS are less divergent along the diagonal line than the scatter plots for SAW. This is one indicator of better goodness of fit when ITOPSIS is used for the multiple criteria aggregation than SAW.

The diagnostics of each response surface model, including R^2, R^2_{Adj}, and Root Mean Square Error (RMSE) in percentage, are listed in Table 5.11. R^2 measures the proportion of the variation explained by the regressed polynomial model, R^2_{Adj} adjusts the R^2 value to make it more comparable over models with different numbers of parameters, and RSME estimates the standard deviation of the random error. The percent RMSE shown in Table 5.11 is normalized by its mean of response.

Higher values of R^2 and R^2_{Adj} and lower values of percent RSME are strong evidences of goodness of fit. It is observed from Table 5.11 that the values of R^2 and R^2_{Adj}, when ITOPSIS is used for the aggregation of the four design criteria, are all higher than when

Table 5.11: The Diagnostics of Response Surface Models for Design Criteria, using ITOPSIS Index and SAW Index as Objective Functions

Diagnostics	OEM	Fuel Mass	Utilization/(Block time)	Passenger Density
ITOPSIS				
R^2	0.975	0.964	0.983	0.957
R^2_{Adj}	0.963	0.951	0.976	0.945
Percent RMSE	1.56%	1.57%	0.54%	0.74%
SAW				
R^2	0.916	0.934	0.973	0.774
R^2_{Adj}	0.9	0.92	0.965	0.743
Percent RMSE	2.58%	2.22%	0.66%	1.84%

SAW is used. The percent RSME, when ITOPSIS is used for the aggregation of the four design criteria, are all lower than when SAW is used. Especially, R^2 of passenger density when ITOPSIS is used is 0.957, while it is only 0.774 when SAW is used. Therefore, it is obtained that the constructed response surface models using ITOPSIS for multiple criteria aggregation are better fitted than using SAW for multiple criteria aggregation. In conclusion, ITOPSIS index is a more appropriate objective function for the optimization framework of incorporating MCDA techniques in aircraft design process than the traditional SAW index.

Model Accuracy Evaluation by Running Additional Data

The accuracy of response surface models when ITOPSIS is used for aggregation are further evaluated by running additional untried data points. The additional untried data points are attached in Appendix D.2. The error analysis between the actual values produced by the original analysis tool (VAMPzero) and the predicted values generated by the response surface models are performed. The mean values and standard deviations of these errors are summarized in Table 5.12. It is found that the means of relative errors for these four design criteria are all less than 0.7% and the standard deviations are less than 2%. The minor errors support that the response surface models predict sufficiently.

In summary, we can conclude that the response surface models can provide adequate approximations to the analysis tool (VAMPzero). The constructed response surface

Table 5.12: Relative Errors Between Actual and Predicted Values for Design Criteria

	OEM	Fuel Mass	Utilization/(Block time)	Passenger Density
Percent μ	0.42%	−0.68%	0.28%	−0.06%
Percent σ	1.80%	1.44%	0.71%	1.00%

models will be further utilized to conduct uncertainty assessment in Section 5.5.

5.5 Uncertainty Assessment for Weighting Factors via Surrogate Models

As noted in Section 5.4, inherent uncertainties and subjectivities of weighting factors have significant impacts on the design solution in the proposed multi-criteria optimization framework. The intractable computation task in uncertainty assessment process is alleviated by the construction of surrogate models. This section presents uncertainty assessment via surrogate models, following the new approach proposed in Chapter 4.

5.5.1 Uncertainty Characterization

As described previously in Section 4.1, uncertainties of weighting factors are described by percentage uncertainties with different confidence levels first. In our case, when the weighting factors are evenly distributed among the four design criteria, the mean value of the weighting factors is

$$\mu_W = [0.25\ 0.25\ 0.25\ 0.25]^{\mathrm{T}}$$

Assume that there exists 20% uncertainty in the weight of OEM with 90% confidence level. In other words, it is 90 percent confident that the weight of OEM would fall within the interval $[w_1 (1 - 20\%), w_1 (1 + 20\%)]$. The percentage untertainties and confidence levels of other design criteria in the weighting factors have similar explanation. The weighting factors with percentage uncertainties and confidence levels are summarized in Table 5.13.

Table 5.13: Uncertainty Characterization for Weighting Factors

	OEM w_1	Fuel Mass w_2	Utilization/(Block time) w_3	Passenger Density w_4
Percentage Uncertainty	20%	30%	20%	10%
Confidence Level	90%	80%	70%	80%

Secondly, percentage uncertainties with different confidence levels are transferred into standard deviations through Equation 4.4 and Equation 4.6, as described in Subsection 4.1.2. For example, the number of standard deviation for w_1 with 20% uncertainty at 90% confidence level, is calclulated by Equation 5.10. The standard deviation for w_1 is calculated by Equation 5.11.

$$\begin{aligned} n_{w_1} &= \sqrt{2} erf^{-1}(\text{Confidence level}) \\ &= \sqrt{2} erf^{-1}(90\%) \\ &= 1.6449 \end{aligned} \tag{5.10}$$

$$\begin{aligned} \sigma_{w_1} &= \frac{\text{Relative error}(\%)\mu_{w_1}}{n_{w_1}} \\ &= \frac{(20\%)(0.25)}{1.6449} \\ &= 0.0304 \end{aligned} \tag{5.11}$$

The same calculation is done for all design criteria. The standard deviation for the weighting factors is

$$\sigma_W = [0.0304\ 0.0585\ 0.0482\ 0.0195]^{\mathrm{T}}$$

In this step, uncertainties of the weighting factors, characterized by percentage uncertainties and confidence levels, are transferred into means and standard deviations. Moreover, μ_W and σ_W are the input for the error propagation calculation step.

5.5.2 Uncertainty Analysis

As discussed in Section 4.2, Monte Carlo-based numerical error propagation technique is applied to propagate uncertainty through surrogate models. 10,000 iterations are performed from normal distribution with parameters μ_W and σ_W. The histograms of the design criteria with uncertainty propagated from the weighting factors via surrogate models are presented in Figure 5.14, where CL stands for confidence level. The mean values and standard deviations of design criteria are also calculated and integrated in the figure. It can be seen that except for fuel mass, the distribution of the propagated uncertainties from the weighting factors for the other three design criteria can be approximately represented by normal distributions.

Robustness Measurement using Signal-to-Noise Ratio

The design criteria with deterministic weighting factors were shown in Table 5.7 in Subsection 5.3.2. The comparison of design criteria with propagated uncertainty from

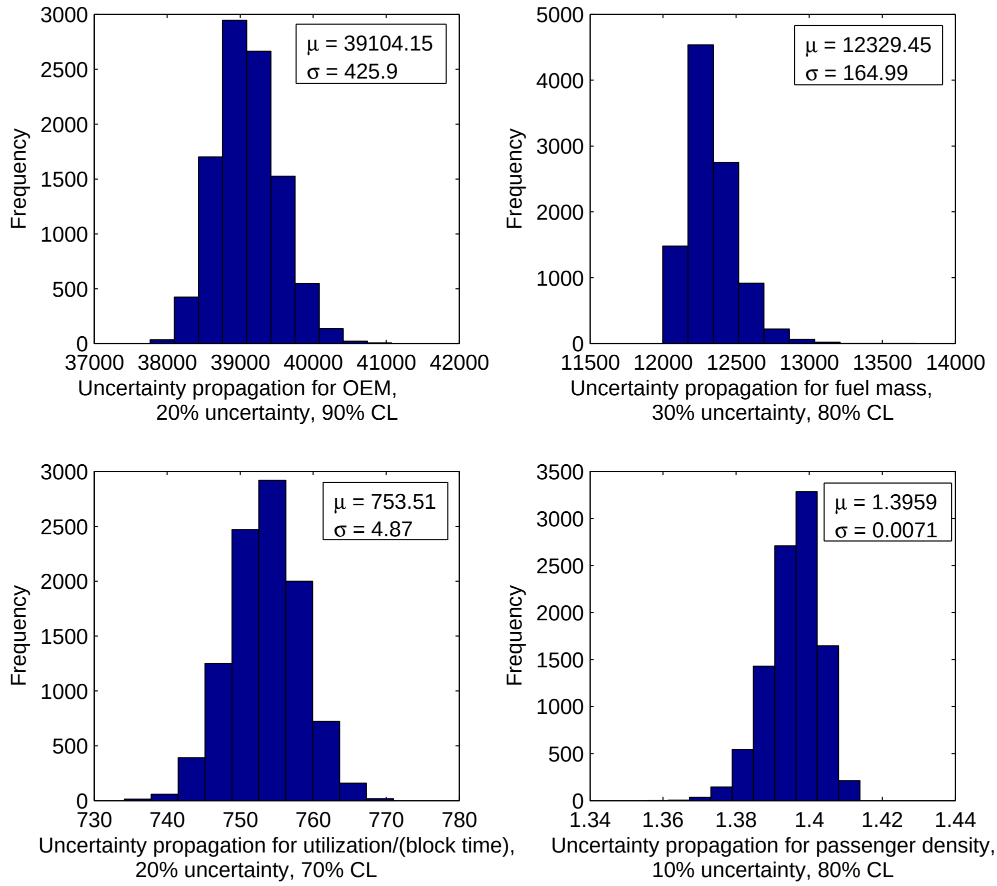


Figure 5.14: Histograms of Uncertainty Propagation for OEM, Fuel Mass, Utilization/(Block time), and Passenger Density

weighting factors and with deterministic weighting factors is summarized in Table 5.14, including means, standard deviations, and SNR (Signal-to-Noise Ratio).

The SNR is calculated according to Equation 4.10 in Subsection 4.2.3 ($\text{SNR} = 20\log_{10}(\frac{\mu}{\sigma})$). Larger SNR value indicates more robustness against uncertainty. For instance, in Table 5.14, 39.26 (dB) means that the magnitude of mean for OEM is $10^{\frac{39.26}{20}} \approx 92$ times the magnitude of its standard deviation. The other SNR values for the other design criteria have similar explanations.

The largest value of SNR for passenger density in Table 5.14 indicates that passenger density is relatively robust to the uncertainty in the weighting factors, while fuel mass is relatively sensitive among the four design design criteria. On one side, the largest

Table 5.14: Comparison of Design Criteria with Deterministic and Uncertain Weighting Factors

Design Criteria	Deterministic Design	Uncertain Design	SNR (dB)
OEM	38705.03	$\mu = 39104.15, \sigma = 425.9$	39.26
Fuel mass	12242.18	$\mu = 12329.45, \sigma = 164.99$	37.47
Utilization/(block time)	751.64	$\mu = 753.31, \sigma = 4.87$	43.79
Passenger density	1.4211	$\mu = 1.3959, \sigma = 0.0071$	45.87

value of SNR for passenger density may be due to the smallest percentage uncertainty assigned in Table 5.13; on the other side, the linearity of passenger density regarding the five design variables, as shown in parametric studies of design criteria in Section 5.1.2, can also leads to highest SNR value of passenger density.

Likewise, the second-higher SNR value of utilization/(block time) ratio among the four design criteria can also be attributed to its linearity with regards to the five design variables. Furthermore, one reason of the smallest SNR for fuel mass probably is also the biggest percentage uncertainty assigned in Table 5.13, another reason can also be attributed to its non-linearity with regards to the five design variables.

Uncertainty Variation in Percentage Uncertainty and Confidence Level

Since uncertainty characterization has substantial impact on the distribution shape and robustness of the design criteria, uncertainty variation in the percentage uncertainty and confidence level are investigated. Especially, the impact behavior of percentage uncertainty is compared with confidence level on the distribution shapes of design criteria.

The percentage uncertainty under investigation ranges from 10%, 30%, and 50%, with confidence level ranges from 10%, 50%, and 90%, as presented in Table 5.15. These percentage uncertainties with different confidence levels are transferred into standard deviations using Equation 4.4 and Equation 4.6, as described in Subsection 4.1.2. It is observed from Table 5.15 that with the same percentage uncertainty, the growth of confidence level reduces the standard deviation of the weighting factors. Likewise, at equal confidence level, the increase of percentage uncertainty leads to higher standard deviation of the weighting factors.

Table 5.15: Uncertainty Variation for Weighting Factors, Regarding Percentage Uncertainty and Confidence Level

	Confidence Level		
Percentage Uncertainty	10%	50%	90%
10%	0.1989	0.0371	0.0152
30%	0.5968	0.1112	0.0456
50%	0.9947	0.1853	0.0760

10,000 Monte Carlo simulations are conducted through the constructed surrogate models for the four design criteria with equal weighting factor μ_W and standard deviation presented in Table 5.15. The distributions for OEM are presented in Figure 5.15, the distributions for the other three design criteria are attached in Appendix C.2. Each row of the figures represents the propagated uncertainty distribution, with specific percentage uncertainty at different confidence levels, while each column represents the propagated uncertainty distribution, with different percentage uncertainties at the same confidence level. Moreover, the mean values and standard deviations of the propagated uncertainties are also integrated into the figures.

These figures serve as graphical confirmation of the analytical analysis obtained from Table 5.15. With the same percentage uncertainty, the increase of confidence level narrows the distribution shape of the design criteria with propagated uncertainty. As an example, the first row in Figure 5.15 indicates that with 10% uncertainty, OEM with propagated uncertainty approaches a normal distribution with lower standard deviation progressively. However, as the increase of percentage uncertainty (the second row and the third row in the figure), the impact behavior of confidence level on the distribution shape of the propagated uncertainty becomes weak. Nevertheless, the increase of the confidence level can reduce the standard deviation effectively, even when substantial percentage uncertainty exists.

Meanwhile, at same confidence level, the increase of percentage uncertainty expands the propagated uncertainty distribution with higher standard deviation. The expansion effect is more severe when there are substantial uncertainties. For instance, the first column in Figure 5.15 shows that at 10% confidence level, the increase of percentage uncertainty enlarges the standard deviation of OEM with propagated uncertainty. With the growth of confidence level (the second column and the third column in the figure), the standard deviation decreases significantly.

Robustness Comparison

In order to measure the robustness of the design criteria against uncertainty in the weighting factors, SNR is also calculated using Equation 4.10 and presented in Figure 5.16. The SNR analysis indicates consistent conclusions previously drawn from the histograms of uncertainty variation. For the same percentage uncertainty, the growth of confidence level lead to the increase of SNR. Since larger SNR indicates more robustness against uncertainty, the robustness of design criteria can be strengthened by the growth of confidence level.

Furthermore, it is also observed from Figure 5.16 that utilization/(block time) and passenger density demonstrate larger SNR than fuel mass and OEM. In other words, utilization/(block time) and passenger density are relatively more robust against the uncertainty in the weighting factors than fuel mass and OEM. Besides, OEM is relatively less robust among the four design criteria.

5.5.3 Sensitivity Analysis

As noted in Section 4.3, sensitivity analysis can identify the relative contribution of input variables to the variability of the model output. Local sensitivity analysis via iterative binary search algorithm and global sensitivity analysis using partial rank correlation coefficients are not followed, since they are established for evaluation decision making problems.

In this study, when MCDA techniques are implemented in design decision making problems, sensitivity analysis can be performed via surrogate models. The prediction profiler in JMP@ provides one effective approach to perform this task. Thus, it is utilized to perform sensitivity analysis for the weighting factors in the aircraft design problem.

In this example, equal weighting factors are assigned to each design criterion, and one linear constraint ($w_1 + w_2 + w_3 + w_4 = 1$) is imposed to the weighting factors. The prediction profilers for the four design criteria are illustrated in Figure 5.17, where the vertical dotted red line for each variable shows its current value, the horizontal dotted red line shows the predicted value of each design criterion for the current values of weighting set. The black lines within the plots show how the predicted value changes when the current value of a variable is changed. The role of the weighting factors in the prediction of the four multiple design criteria can be visualized, by moving the vertical dotted line or by directly entering variable value.

The steepness of the prediction trace can reflect the sensitivity of variables. It can be observed from Figure 5.17 that the prediction traces on the diagonal line have the steepest slopes. In other words, they are the most sensitive variables for the predicted criteria on each row using the fitted response surface model. This is consistent with physical explanation. For instance, w_1 has the most steepest negative gradient in the first row when predicting OEM, considering that w_1 is the weighting factors for OEM during optimization, thus, OEM will decrease with the increase of w_1. Based on the same token, w_2 is the most sensitive variable in predicting fuel mass, w_3 is the most sensitive variable in predicting utilization/(block time) ratio, and w_4 is the most sensitive variable in predicting passenger density.

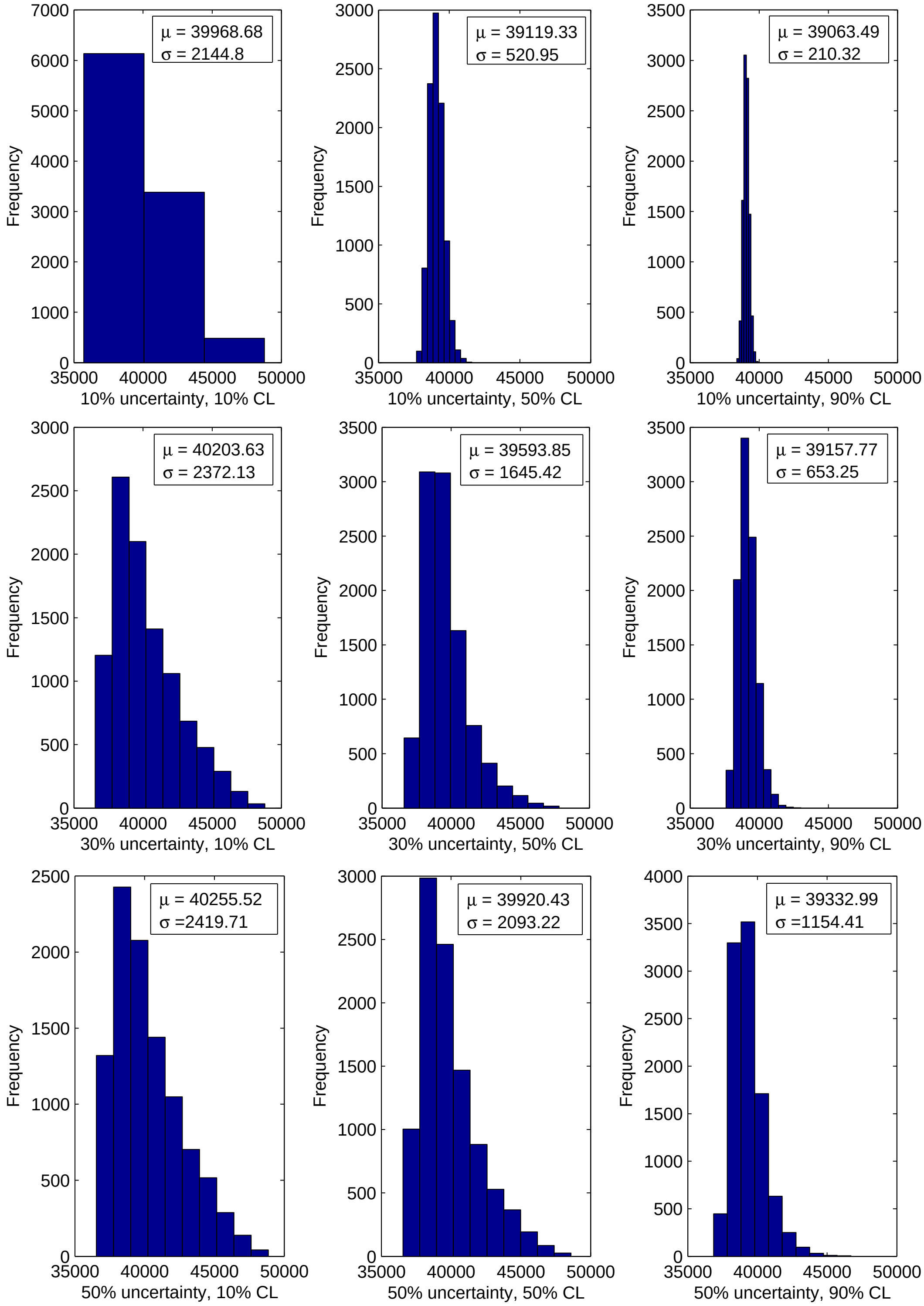


Figure 5.15: Uncertainty Variation for OEM

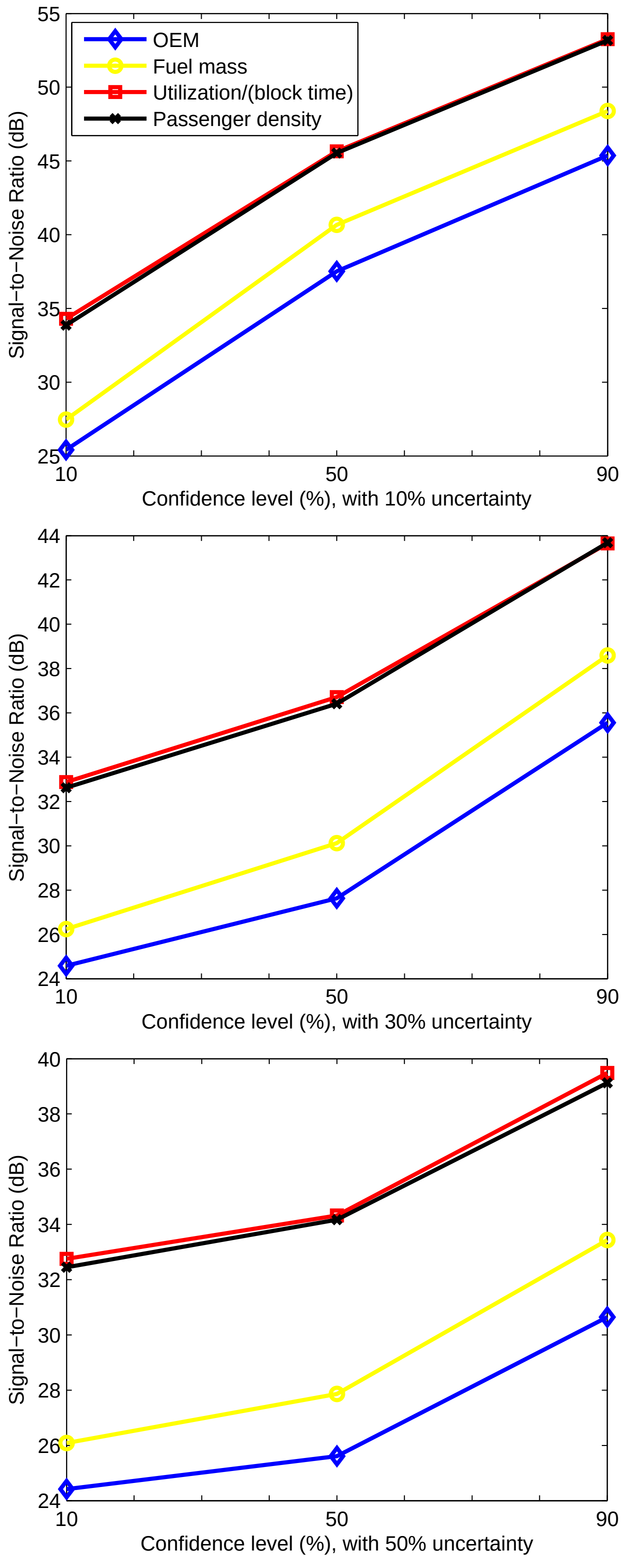


Figure 5.16: Robustness Comparison for Four Design Criteria

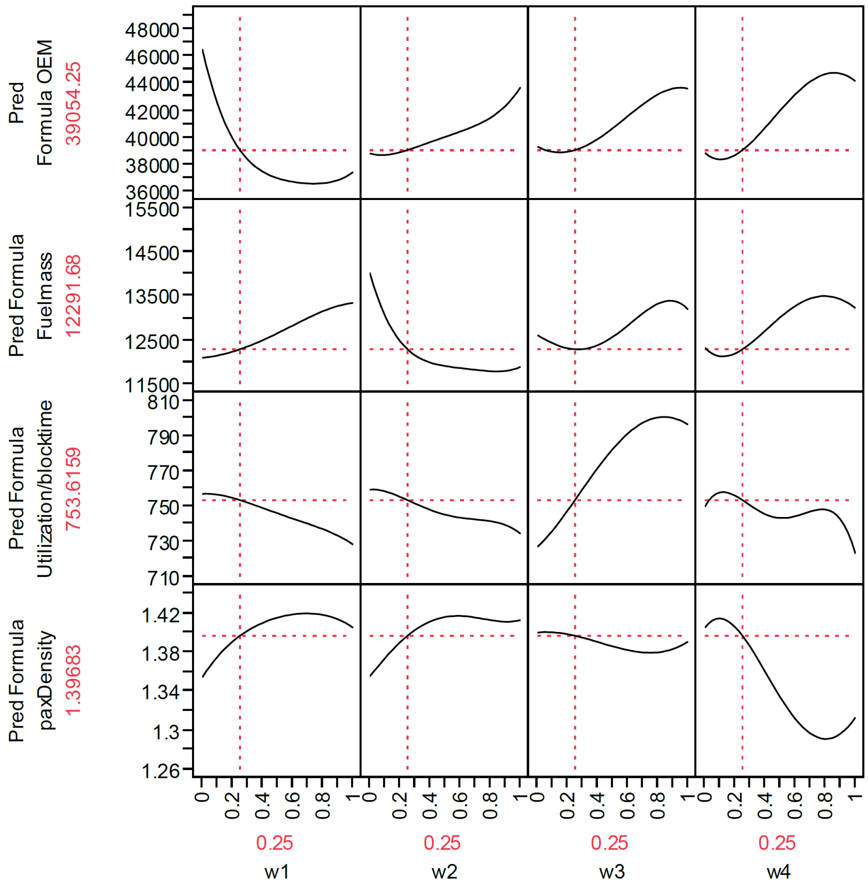


Figure 5.17: The Prediction Profilers for Four Design Criteria

5.6 Discussion

This chapter explored the feasibility and assessed the added values of applying MCDA techniques in aircraft design problems. A new optimization framework incorporating MCDA techniques for aircraft conceptual design process was established. The developed intelligent multi-criteria decision support system was used to select an appropriate MCDA method. It was demonstrated that the chosen MCDA method with improvement (ITOPSIS) provides a better objective function for the optimization than the traditional weighted sum (SAW) method. Furthermore, considering that inherent uncertainties and subjectivities of weighting factors have crucial impacts on the design solution, surrogate models for the multiple design criteria in terms of weighting factors were constructed. Results show that the constructed surrogate models can enable efficient uncertainty assessment for the weighting factors.

In this section, optimization algorithms used in aircraft design are discussed, followed by surrogate model development for design criteria in terms of weighting factors.

Optimization Algorithms in Aircraft Design

As noted in Subsection 5.3.1, there are several optimization algorithms currently available, among which gradient-based methods and genetic algorithms are most widely used in aircraft design. Which optimization method to use depends on the optimization problem under consideration.

The choice of gradient-based methods for the proposed optimization framework was based on the parametric studies performed in Subsection 5.1.2, where all design variables under investigation were continuous, and the objective functions with respect to the design variables in the conceptual aircraft design tool were rather smooth.

Furthermore, the focus of this research has been on developing the framework of incorporating MCDA techniques in aircraft design process, particularly on exploring the feasibility and assessing the added values, not on the optimization itself. A hybrid optimizer combining genetic algorithm and gradient-based method could be also used in order to provide a more global optimization and include discrete design variables. However, this is beyond the scope of this study and can be regarded as future research.

Surrogate Model Constructions for Design Criteria in terms of Weighting Factors

As noted in Section 5.4, there are typically four steps in constructing the surrogate models: experimental design, model choice, model fitting, and model validation (30).

The choice of experimental design has a critical impact on the accuracy of the surrogate models. In this study, the experimental designs for the weighting factors have to satisfy that for each experimental run, the sum of the factor settings equals to 1. One modified LHS with Dirichlet distribution was employed, as presented in Subsection 5.4.1. Other sampling strategies with space filling properties could be also investigated.

Response surface model was utilized to construct the surrogate models in the model choice step. Furthermore, Kriging models are alternative techniques to construct surrogate models with more sound statistical meaning (71). Kriging models interpolate the observed data and fit the model using maximum likelihood estimate.

A comparison of response surface model and Kriging model for multidisciplinary design optimization was presented by (75), with the application to the design of an aerospike nozzle. The authors concluded that the second-order response surface models and Kriging models using a constant underlying global model and a Gaussian correlation function yielded comparable results. Besides, it was stated that the choice of the modeling technique depends on the expectations of what the underlying response might look like (30). Future research can be done about using Kriging model to construct the surrogate models.

6

Proof of Concept 2: MCDA in Aircraft Evaluation

In this chapter, the application of appropriate MCDA techniques in aircraft evaluation decision making process is demonstrated, following a three-steps framework: definition of the decision making problem, selection of the most appropriate MCDA method, and uncertainty assessment in the decision analysis process.

The chapter is organized as follows. Section 6.1 defines the business aircraft evaluation problem. Section 6.2 presents the selection of the most appropriate MCDA method, through the developed intelligent multi-criteria decision support system, as described in Chapter 3. Section 6.3 presents the results of applying the appropriate MCDA method in the business aircraft evaluation problem. Section 6.4 presents uncertainty assessment in decision analysis process, according to the new approach proposed in Chapter 4. Section 6.5 discusses the implementation of MCDA techniques in aircraft evaluation problems.

6.1 Definition of the Decision Making Problem

Assume that one business aviation customer needs to purchase a business jet. At present, there are six major business jet manufacturers: Canadian Bombardier, American Cessna, French Dassault, Brazilian Embraer, American Gufstream, and American Hawker. There are five different segments for different types of the product models: very light jets, light jets, medium jets, large jets, and large corporate airliners. The segmentation is primarily determined by a combination of price, range, and cabin volume, as summarized in Table 6.1.

Table 6.1: Segmentation Criteria for Business Jets (13)

Business Aircraft Segmentation	**Price** ($ Millions)	**Range** (km)	**Cabin volume** (m^3)
Very light jets	< 7	< 3148	< 8.5
Light jets	7 - 18	3148 - 5741	8.5 - 19.8
Medium jets	18 - 42	5741 - 9260	19.8 - 42.5
Large jets	46 - 68	> 9260	42.5 - 85.0
Large corporate airliners	> 68	> 9260	> 85

The six major business jet manufacturers are briefly introduced as follows. Bombardier offers three families of business jets: Learjet, Challenger, and Global. Cessna mainly offers light to medium size business aircrafts. Dassault produces medium to large size business jets. Embraer offers five product models of business jets, ranging from light to large size aircrafts. Gulfstream offers light, medium, and large business aircrafts. Hawker produces mainly light and medium business jets. In addition, Airbus and Boeing also offer Airbus Corporate Jet (ACJ) and Boeing Business Jet (BBJ), based on their A319 and B737 series, respectively. These large size aircrafts are most expensive in the business jet market.

There are more than forty different types of business aircraft available in the current market, costing from $ 1 million up to almost $ 100 hundred million. How to choose the appropriate aircraft to meet the needs of the business aviation customer is a complicated decision making process. In addition to costs, there are several other criteria to be evaluated at the same time. For instance, aircraft configuration, aircraft performances, environmental aspects, and several additional attributes. Therefore, considering these multiple conflicting criteria simultaneously, the evaluation and selection of a business jet is a typical MCDA problem and needs to be prudently conducted.

In the following subsections, the identification of the evaluation criteria for business aircraft is discussed first, followed by the quantification of additional soft criteria.

6.1.1 Identification of Evaluation Criteria

The specifications of business aircraft are presented in Figure 6.1. Based on the specifications, the evaluation criteria for business aircraft can be categorized into four groups:

- **Economic criteria**: purchase price and operating costs.
- **Performance criteria**: maximum payload, maximum range, cruise speed, fuel consumption, and take-off field length.
- **Environmental criteria**: noise and CO2 emissions.
- **Additional soft criteria**: passenger comfort level, product support level, and manufacturers reputation.

We are confronted with the same question as in Subsection 5.1.1: Which evaluation criteria are most appropriate to be fed into the MCDA method for the business aircraft evaluation problem? In order to better answer this question, the quantification of additional soft criteria is presented first, followed by the determination of which evaluation criteria would be further feed into the MCDA method.

6.1.2 Quantification of Additional Soft Criteria

Among these four groups, the additional soft criteria are considered to be the decisive factors in the business aircraft evaluation problem. However, these soft criteria cannot be fed into the MCDA method directly without quantification. In this subsection, the quantification of passenger comfort level, product support level, and manufacturer's reputation are presented, respectively.

Quantification of Passenger Comfort Level

Passenger comfort level can be influenced by several factors, for instance, space utilization, cabin noise, and vibration. Among these factors, space utilization is known as predominant for passenger comfort, thus, we will focus on space utilization in this research. The passenger seating configuration, cabin height, cabin width, cabin length,

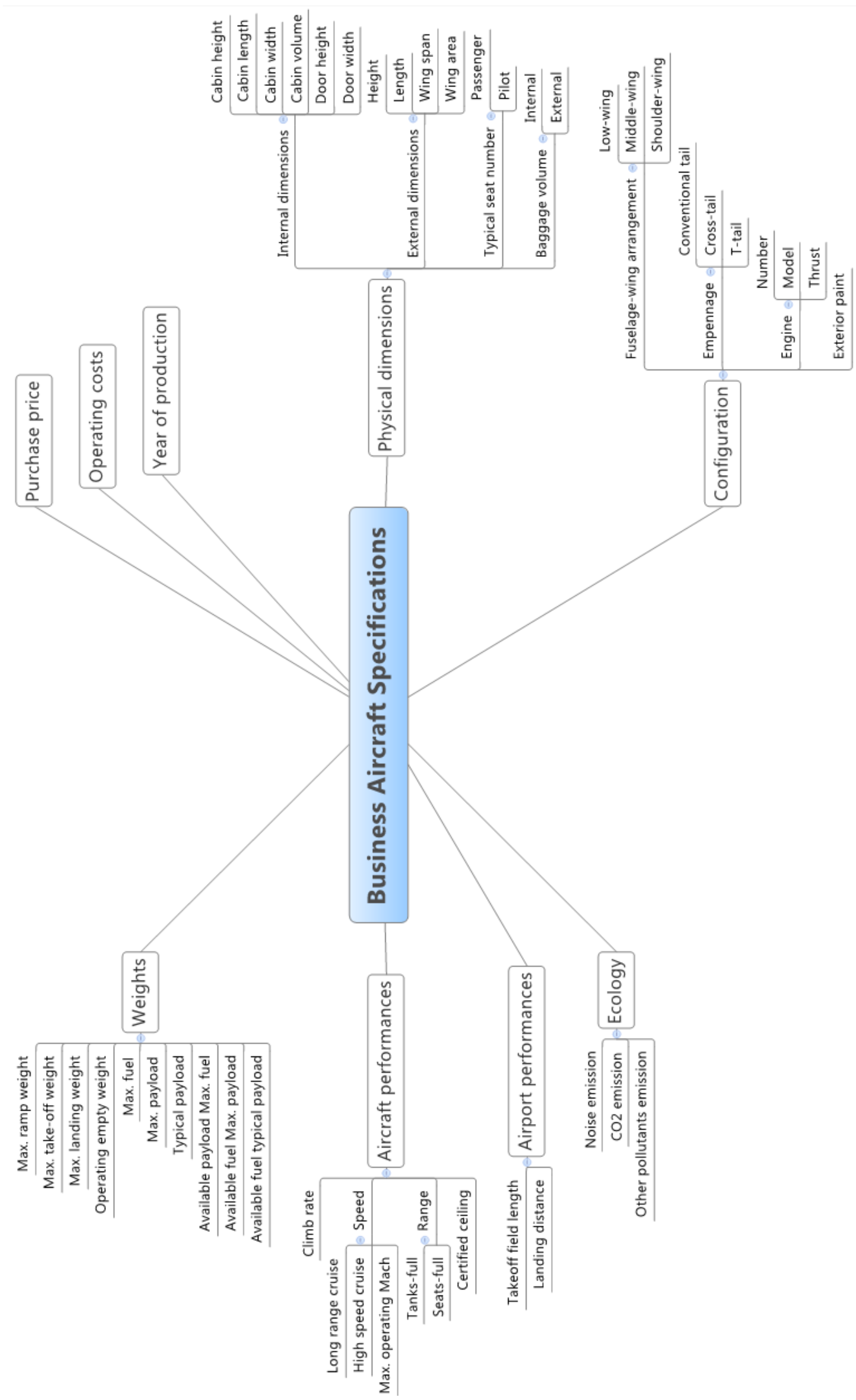


Figure 6.1: The Specifications of Business Aircraft (2)

and cabin volume determine the space utilization. The passenger comfort level can be quantified by cabin volume per passenger (m^3/pax), as calculated in Equation 6.1.

$$\text{Cabin volume per passenger} = \frac{\text{Cabin volume}}{\text{Typical passenger seat number}} \tag{6.1}$$

Quantification of Product Support Level

Product support level is quantified based on the aviation international news 2010 product support survey (80). The product support survey is conducted entirely on the Internet, qualified readers are asked to rate their business aircrafts, engines, and avionics in ten categories. The ten categories are summarized in Table 6.2, where the explanations of key points that the survey participants were asked to consider are also included. The rating scale ranges from 1 (inadequate) to 10 (excellent), as illustrated in Figure 6.2.

Table 6.2: Ten Categories of the Aviation International News 2010 Product Survey (80)

Categories	**Explanations of Key Points**
1. Authorized Service Center	Estimated cost versus actual cost, on-time performance, scheduling ease, and service experience.
2. Factory Service Center	The same as with the authorized service center.
3. Parts Availability	In stock versus back order and shipping time.
4. Costs of Parts	Value for price paid.
5. Aircraft On Ground Response	The speed, accuracy, and cost to get a grounded aircraft back in the air as soon as possible.
6. Warranty Fulfillment	Ease of paperwork and extent of coverage.
7. Technical Manuals	Ease of use, formats available, timeliness of updating.
8. Technical Representatives	Response time, knowledge, and effectiveness.
9. Maintenance Tracking Programs	Cost, ease of use, accuracy, and reliability.
10. Overall Aircraft Reliability	Product's overall reliability and quality against the competition's.

The 2010 product survey invited 17,284 readers to participate and 921 completed the survey, with a return rate of 5.3%. The results of the 2010 product survey are presented in Figure 6.3, where the aircraft are listed in the order of their overall average scores. The newer business jets are less than ten years old, and the older business jets are more than ten years old. The bold number indicates the highest number in each category.

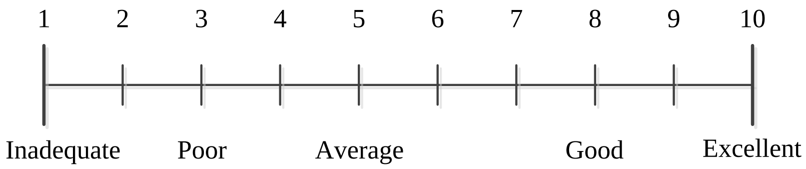


Figure 6.2: Rating Scale of the Aviation International News 2010 Product Survey (80)

	Overall Average 2010	Auth. Service Centers	Factory Service Centers	Parts Availability	Cost of Parts	AOG Response	Warranty Fulfillment	Technical Manuals	Technical Reps	Mx Tracking Programs	Overall Aircraft Reliability
NEWER BUSINESS JETS											
Gulfstream *(GIV through G550)*	**8.31**	7.96	8.24	**8.44**	6.35	**8.58**	**8.55**	**8.24**	**8.74**	**8.71**	**9.08**
Cessna *(Citation)*	8.22	7.78	8.28	8.33	**6.93**	8.43	8.40	8.04	8.56	8.41	8.79
Bombardier *(Learjet)*	7.95	7.69	7.78	7.90	6.51	8.13	8.41	7.91	8.82	8.00	8.38
Gulfstream *(G100 to G200)*	7.75	7.70	7.54	7.32	6.60	7.81	8.07	7.84	8.26	7.80	8.38
Dassault *(Falcon)*	7.68	7.55	6.94	7.71	6.27	8.07	8.07	7.56	7.97	8.03	8.47
Hawker Beechcraft *(Hawker except 400XP)*	7.66	7.84	7.90	7.07	6.23	7.28	8.27	7.31	8.24	8.25	8.26
Bombardier *(Challenger)*	7.63	7.65	7.48	7.24	6.08	7.72	7.87	7.66	8.29	7.90	8.30
Hawker Beechcraft *(Premier I, Hawker 400XP)*	7.41	**8.11**	**8.29**	7.13	5.52	7.18	7.27	7.62	7.59	7.76	7.92
Bombardier *(Global Express/XRS, Global 5000)*	7.16	7.40	7.16	6.77	5.63	7.18	7.39	6.88	8.20	7.44	7.52
OLDER BUSINESS JETS											
Gulfstream *(GII through GIV)*	**8.14**	**7.96**	**7.76**	**8.35**	**6.14**	**8.65**	**8.27**	**8.23**	**8.61**	**8.61**	**8.76**
Dassault *(Falcon)*	7.59	7.91	6.97	7.73	5.61	7.92	7.35	7.33	8.11	7.91	8.84
Cessna *(Citation)*	7.46	7.58	7.08	7.70	5.86	7.53	7.22	7.70	7.58	7.79	8.32
Bombardier *(Learjet)*	7.35	7.32	6.58	7.34	6.00	7.67	7.04	7.64	7.74	7.92	8.01
Hawker Beechcraft *(Hawker)*	7.18	7.33	6.89	6.92	5.62	6.98	6.87	7.27	7.93	7.95	8.06
Bombardier *(Challenger)*	7.06	7.79	6.86	6.71	5.29	7.03	6.76	7.16	7.45	7.74	7.93
Hawker Beechcraft *(Premier I, Diamond, Beechjet 400A)*	6.94	7.25	7.25	6.95	5.58	7.06	7.00	6.26	6.77	7.33	8.05

Figure 6.3: Results of the Aviation International News 2010 Product Survey (80)

According to the survey results shown in Figure 6.3, the product support level of Gulfstream ranked first for both newer and older business jets in 2010. Moreover, among the ten categories summarized in Table 6.2, the overall aircraft reliability and quality received the highest score and the technical representatives received the second-highest score for all the manufacturers for both newer and older business jets, with contrast to that the cost of parts received the lowest score.

Quantification of Manufacturer's Reputation

Manufacturer's reputation is quantified according to the aviation week's 16th annual top-performing companies study for 2010 (4). The top-performing companies study was launched in 1996 by *Aviation Week & Space Technology*, with the purpose of assessing the operational performance of publicly traded companies in the aerospace and defense industries. The company ranking is based on a composite scoring of four equally weighted performance categories. The scores range from 1 (worst performance) to a maximum value 99 (best performance). The four categories are summarized in Table 6.3.

Table 6.3: Four Categories of the Aviation Week's 16th Annual Top-Performing Companies Study (4)

Categories	**Measurement**
1. Return on Invested Capital	Investment decisions, companies with superior operating profit are rewarded.
2. Earning Momentum	Earning quality and revenue expansion.
3. Asset Management	Efficiency in employing the resources.
4. Financial Health	Overall solvency and available liquidity.

For the purpose of this study, the scores for the six major business jet manufacturers are presented in Table 6.4. It should be noted that Cessna, Gulfstream, and Hawker are not explicitly on the list of the top-performing companies study. Thus, the scores of their parent companies are used instead. According to the scores shown in Table 6.4, a higher score in the top-performing companies study represents better reputation. Thus, Gulfstream has the highest reputation, while Cessna has the lowest reputation.

In summary, in the additional soft criteria group, passenger comfort level is quan-

Table 6.4: The Scores of the Six Major Business Jet Manufacturers (4)

Manufacturers	**Scores**
Bombardier	55
Cessna (Textron)	39
Dassault	74
Embraer	60
Gulfstream (General Dynamics)	82
Hawker (Raytheon)	78

tified by cabin volume per passenger (m^3/pax), product support level is quantified according to the overall average scores obtained via the aviation international news 2010 product survey, as shown in Figure 6.3, and manufacturer's reputation is quantified based on the aviation week's 16th annual top-performing companies study for 2010, as summarized in Table 6.4.

Determination of Evaluation Criteria

Empirical studies in consumer behavior and industrial market context have shown that the quality of a decision has an inverted U-shaped relationship with the number of alternatives, and the number of intensively discussed alternatives is less than five (31). In practice, a small number of alternatives can be obtained by a simple check-list of desirable features (84).

In this business aircraft evaluation problem, typical passenger seat number, maximum range, and purchase price, are utilized as filter criteria for initial screening in the first phase of decision making process. The filter criteria can highly facilitate the business aircraft evaluation problem by reducing the number of alternatives under consideration.

Furthermore, the operating costs will not be fed into the MCDA method, the reasons are listed as follows. The operating costs are composed of fixed costs and variable costs. Fixed costs are irrespective of aircraft utilization, and thus include insurance, training costs, and other miscellaneous costs. Variable costs vary with aircraft utilization, consisting of fuel costs, maintenance costs, and miscellaneous trip expenses. Fixed costs are directly proportional to the purchase price, while variable costs are directly related to fuel consumption. Additionally, CO2 emission is also largely fuel-based.

Thus, instead of using operating costs as an independent evaluation criterion, aircraft purchase price is used to approximate the fixed operating costs, and fuel consumption is utilized as a proxy for the variable operating costs and CO2 emission.

In summary, three filter criteria and seven decision criteria which will be fed into the MCDA method are summarized in Table 6.5, where EPNdB represents the decibels of Effective Perceived Noise.

Table 6.5: Ten Evaluation Criteria of Business Aircraft

	Name	Units
Filter Criteria	Typical passenger seat number	pax
	Maximum range	km
	Purchase price	$ Millions
Decision Criteria	Fuel consumption per seat kilometer	kg/pax/km
	High-speed cruise speed	km/h
	Take-off field length	m
	Noise	EPNdB
	Cabin volume per passenger	m^3/pax
	Product support level	-
	Manufacturer's reputation	-

One Scenario for Business Aviation Customer

Assume that one business aviation customer considers to purchase a business jet with 8 to 10 typical passengers on board. The aircraft range with maximum fuel and available payload should be around 5500 km to 6500 km, and the purchase price is between $ 20 millions and $ 25 millions.

In the available business jet market, four business jet alternatives satisfy the needs of the customer. The values of the three filter criteria and the seven decision criteria for the four business jet alternatives are summarized in Table 6.6.

In Table 6.6, maximum range is when the aircraft is with full fuel and maximum available payload, and with the National Business Aviation Association (NBAA) Instrument Flight Rules (IFR) fuel reserves (370.4 km or 200 nm alternate). Purchase price is *Business & Commercial Aviation (BCA)* equipped price published in May 2011 issue (2). Fuel consumption is calculated based on the fuel used for the mission of flying

Table 6.6: The Values of Evaluation Criteria for the Four Business Jet Alternatives

	Alternatives			
	A_1 Bombardier Challenger 300	A_2 Cessna Citation X	A_3 Gulfstream G200	A_4 Hawker H4000
				
Filter Criteria				
F_1: Typical passenger seat number	8	9	10	8
F_2: Maximum range (km)	5975	5656	6378	5808
F_3: Purchase price ($ Millions)	24.7500	21.6330	23.3250	22.9089
Decision Criteria				
C_1: Fuel consumption per seat kilometer (kg/pax/km)	0.2396	0.2720	0.2264	0.2624
C_2: High-speed cruise speed (km/h)	870	952	870	870
C_3: Take-off field length (m)	1466	1567	1854	1545
C_4: Noise (EPNdB)	84.2333	82.4333	86.7333	86.1000
C_5: Cabin volume per passenger (m^3/pax)	4.0500	2.3556	3.1000	3.4375
C_6: Product support level	7.63	8.22	7.75	7.66
C_7: Manufacturer's reputation	55	39	82	78

1852 km (1000 nm) with four passengers on board. Noise is calculated by the average value of take-off, sideline, and approach noise. It should be noted that only the seven decision criteria (from C_1 to C_7) will be further fed into the MCDA method.

6.2 Selection of an Appropriate MCDA Method

The selection of the most appropriate MCDA method for the business aircraft evaluation problem is presented in this section, through the developed intelligent multi-criteria decision support system, as discussed in Chapter 3. The user guide can be found in Appendix B. The step by step problem solving process is explained and discussed in the following subsections.

Step 1: Define the Problem

As discussed in Section 6.1, the objective of this decision making problem is to evaluate the performance of the business jets and identify which one has the best compromised performance using one appropriate MCDA method. The developed intelligent multi-criteria decision support tool will be employed to facilitate this decision making process.

Step 2: Define the Evaluation Criteria

With the purpose of identifying the most appropriate method, sixteen widely used MCDA methods are studied and their characteristics are stored in a method database. To compare the appropriateness of the methods with respect to the given problem, each method is assessed based on the proposed twelve evaluation criteria. The twelve evaluation criteria are captured by answering twelve questions, as shown in Figure 6.4.

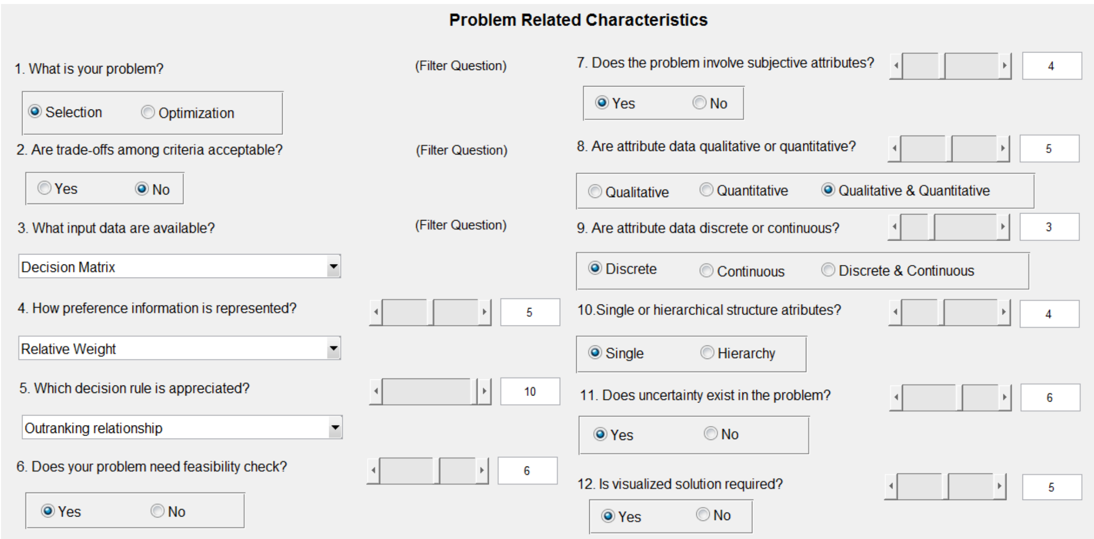


Figure 6.4: Questions Related to Evaluation Criteria for Method Selection in Business Aircraft Evaluation Process

Step 3: Perform Initial Screening

The infeasible MCDA methods are eliminated by the three filter questions. For the business aircraft evaluation problem, with the assumption that trade-offs between criteria are not permitted, all compensatory methods are excluded and only non-compensatory methods will be further used to solve this problem.

Step 4: Define the Preferences on Evaluation Criteria

When selecting the most appropriate method, the DM's preference information on the evaluation criteria can be defined using slide bars in the integrated user interface, where 0 stands for extremely unimportant and 10 represents extremely important. In this example, decision rule is considered most important, thus, 10 is assigned to this scoring question.

Step 5: Calculate the Appropriateness Index

The match of a particular method and the given problem is quantified by the Appropriateness Index (AI). In this step, AI for each of the MCDA methods is calculated by using Equation 3.1, as described in Subsection 3.2.5 in Chapter 3.

Step 6: Evaluate the MCDA methods

According to Step 5, the AI of the MCDA methods are obtained and presented in Figure 6.5, where higher score represents more appropriateness of the method when solving the given problem.

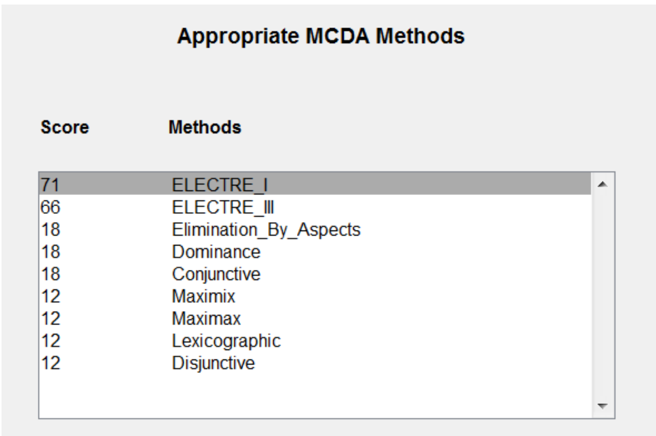


Figure 6.5: MCDA Methods Ranking List in Business Aircraft Evaluation Process

Step 7: Choose the Most Suitable Method

As shown in Figure 6.5, ELECTRE I gets the highest score among the MCDA methods, therefore, it is selected as the most appropriate method to solve the business aircraft evaluation problem. The mathematical calculation steps are built in the MATLAB-based decision support system, thus, the DM can simply click the name of the method and the methodology instruction of ELECTRE I will be displayed to guide the DM to solve the given problem and get the final solution, as illustrated in Figure 6.6. The evaluation results using ELECTRE I will be presented in Section 6.3.

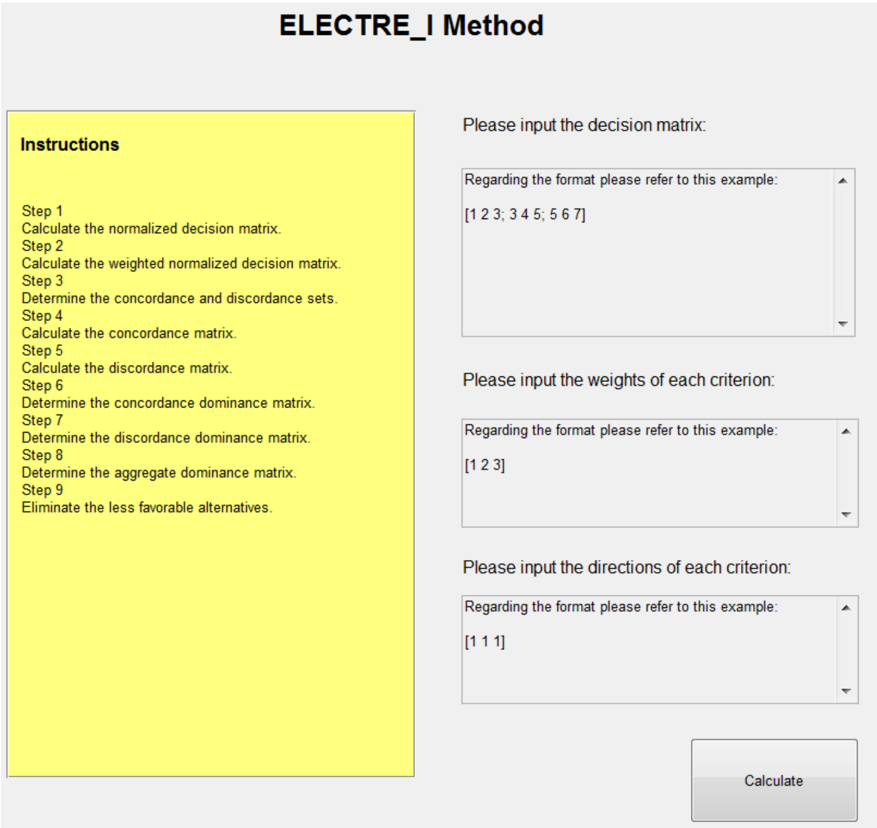


Figure 6.6: Methodology Instructions for ELECTRE I

Step 8: Conduct Sensitivity Analysis

The answers to the twelve questions and the DM's preference information can be varied in the MCDA method selection process. In our integrated user friendly interface, the

DM can adjust the weights of each criterion by moving the corresponding slide bars. If the DM is satisfied with the final results, the solution can be implemented. Otherwise, the DM can go back to Step 2 and modify the input data or preference information and repeat the selection process until a satisfying outcome is obtained.

In this example, with the current preference information and input data, it is observed from Figure 6.5 that ELECTRE III is ranked second by the multi-criteria decision support system. ELECTRE III is an extended version of ELECTRE I, where an indifference threshold, a preference threshold, and a veto threshold have to be defined for each criterion. These extra 21 threshold values for the seven evaluation criteria increase the complexity of the business aircraft evaluation problem significantly. Moreover, these extra 21 threshold values are rather subjective and different DMs often have different threshold values. Therefore, ELECTRE I, will be further used to solve the business aircraft evaluation problem.

6.3 Evaluation Results using ELECTRE I

When ELECTRE I is utilized to solve the business aircraft evaluation problem, it requires a decision matrix as input data and weighting factors as the presentation of DM's preference information. The decision matrix is shown in matrix D, where each row corresponds to one business jet alternative, and each column corresponds to one decision criterion. In the first step of evaluation, equal weighting factors are considered, as shown in vector W.

$$D = \begin{bmatrix} 0.2396 & 870 & 1466 & 84.2333 & 4.0500 & 7.63 & 55 \\ 0.2720 & 952 & 1567 & 82.4333 & 2.3556 & 8.22 & 39 \\ 0.2264 & 870 & 1854 & 86.7333 & 3.1000 & 7.75 & 82 \\ 0.2624 & 870 & 1545 & 86.1000 & 3.4375 & 7.66 & 78 \end{bmatrix}$$

$$W = [0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.14290.1429\ ]^{\mathrm{T}}$$

The stepwise calculations of ELECTRE I are presented in detail in the following subsection, based on the methodology description in Subsection 2.2.4 in Chapter 2.

6.3.1 Stepwise Calculations of ELECTRE I

There are two kinds of criteria: benefit criteria and cost criteria. The DM prefers bigger values for benefit criteria and smaller values for cost criteria. In the business aircraft evaluation problem, benefit criteria are high-speed cruise speed (C_2), cabin volume per passenger (C_5), product support level (C_6), and manufacturer's reputation (C_7), while fuel consumption per seat kilometer (C_1), take-off field length (C_3), and noise (C_4) are cost criteria. Before conducting the normalization, the cost criteria are transformed into benefit criteria by taking the reciprocal values.

1. Normalize the decision matrix D.

$$D_n = \begin{bmatrix} 0.5178 & 0.4881 & 0.5423 & 0.5035 & 0.6149 & 0.4879 & 0.4175 \\ 0.4561 & 0.5341 & 0.5073 & 0.5145 & 0.3577 & 0.5257 & 0.2960 \\ 0.5480 & 0.4881 & 0.4288 & 0.4890 & 0.4707 & 0.4956 & 0.6225 \\ 0.4728 & 0.4881 & 0.5145 & 0.4926 & 0.5219 & 0.4899 & 0.5921 \end{bmatrix}$$

2. Calculate the weighted normalized decision matrix D_{nw}.

$$D_{nw} = \begin{bmatrix} 0.0740 & 0.0697 & 0.0775 & 0.0720 & 0.0879 & 0.0697 & 0.0597 \\ 0.0652 & 0.0763 & 0.0725 & 0.0735 & 0.0511 & 0.0751 & 0.0423 \\ 0.0783 & 0.0697 & 0.0613 & 0.0699 & 0.0673 & 0.0708 & 0.0890 \\ 0.0676 & 0.0697 & 0.0735 & 0.0704 & 0.0746 & 0.0700 & 0.0846 \end{bmatrix}$$

3. Determine the concordance and discordance sets.

   For instance, for the pair of alternatives A_1 and A_2, the set of decision criteria is divided into two disjoint subsets. The concordance set C_{12} is composed of all criteria which support that A_1 is preferred to A_2. The discordance set D_{12} is the complementary set of the concordance set C_{12}, with respect to the decision criteria set $\{1, 2, 3, 4, 5, 6, 7\}$.

$$C_{12} = \{1, 3, 5, 7\} \qquad D_{12} = \{2, 4, 6\}$$

$$C_{13} = \{2, 3, 4, 5\} \qquad D_{13} = \{1, 6, 7\}$$

$$C_{14} = \{1, 2, 3, 4, 5\} \qquad D_{14} = \{6, 7\}$$

$$C_{21} = \{2, 4, 6\} \qquad D_{21} = \{1, 3, 5, 7\}$$

$$C_{23} = \{2, 3, 4, 6\} \qquad D_{23} = \{1, 5, 7\}$$

$$C_{24} = \{2, 4, 6\} \qquad D_{24} = \{1, 3, 5, 7\}$$

$$C_{31} = \{1, 2, 6, 7\} \qquad D_{31} = \{3, 4, 5\}$$

$$C_{32} = \{1, 5, 7\} \qquad D_{32} = \{2, 3, 4, 6\}$$

$$C_{34} = \{1, 2, 6, 7\} \qquad D_{34} = \{3, 4, 5\}$$
$$C_{41} = \{2, 6, 7\} \qquad D_{41} = \{1, 3, 4, 5\}$$
$$C_{42} = \{1, 3, 5, 7\} \qquad D_{42} = \{2, 4, 6\}$$
$$C_{43} = \{2, 3, 4, 5\} \qquad D_{43} = \{1, 6, 7\}$$

4. Calculate the concordance matrix $M_{concordance}$.

   The concordance index is calculated by the sum of criteria weights which are contained in the concordance set. For example, the concordance index c_{12} between A_1 and A_2 is calculated by Equation 6.2.

$$M_{concordance} = \begin{bmatrix} - & 0.5716 & 0.5716 & 0.7145 \\ 0.4287 & - & 0.5716 & 0.4287 \\ 0.5716 & 0.4287 & - & 0.5716 \\ 0.4287 & 0.5716 & 0.5716 & - \end{bmatrix}$$

$$c_{12} = \frac{\sum\limits_{j \in C_{12}} w_j}{\sum\limits_{j=1}^{7} w_j} = w_1 + w_3 + w_5 + w_7 = 0.5716 \tag{6.2}$$

5. Calculate the discordance matrix $M_{discordance}$.

   The discordance index reflects the degree to which one alternative is worse than the other. For instance, the discordance index d_{12} between A_1 and A_2 is calculated by Equation 6.3.

$$M_{discordance} = \begin{bmatrix} - & 0.1793 & 1.0000 & 1.0000 \\ 1.0000 & - & 1.0000 & 1.0000 \\ 0.7038 & 0.2406 & - & 1.0000 \\ 0.5327 & 0.1554 & 0.8767 & - \end{bmatrix}$$

$$\begin{aligned} d_{12} &= \frac{\max\limits_{j \in D_{12}} |v_{1j} - v_{2j}|}{\max\limits_{j \in (1,2,\ldots,7)} |v_{1j} - v_{2j}|} \\ &= \frac{\max\{0.0066, 0.0016, 0.0054\}}{\max\{0.0088, 0.0066, 0.0050, 0.0016, 0.0368, 0.0054, 0.0174\}} \\ &= \frac{0.0066}{0.0368} \\ &= 0.1793 \end{aligned} \tag{6.3}$$

6. Determine the concordance dominance matrix $M_{concordance\ dominance}$.

   A concordance threshold c needs to be chosen to perform the concordance test. In this study, the average value of the elements in the concordance matrix $M_{concordance}$ is used, $c = c_{avg} = 0.5359$.

   For instance, A_1 possibly dominates alternative A_2, if $c_{12} \geq c$. In this example, $c_{12} \geq c$ (0.5716 $\geq$ 0.5359), thus, the concordance test is passed and the concordance dominance index is 1. Otherwise, if the concordance test is failed, the concordance dominance index is 0.

$$M_{concordance\ dominance} = \begin{bmatrix} - & 1 & 1 & 1 \\ 0 & - & 1 & 0 \\ 1 & 0 & - & 1 \\ 0 & 1 & 1 & - \end{bmatrix}$$

7. Determine the discordance dominance matrix $M_{discordance\ dominance}$.

   A discordance threshold d needs to be chosen to perform the discordance test. In this study, the average value of the elements in the discordance matrix $M_{discordance}$ is used, $d = d_{avg} = 0.7240$.

   For instance, A_1 possibly dominates A_2, if $d_{12} \leq d$. In this example, $d_{12} \leq d$ (0.1793 $\leq$ 0.7240), thus, the discordance test is passed and the discordance dominance index is 1. Otherwise, the discordance dominance index is 0 when the discordance test is failed.

$$M_{discordance\ dominance} = \begin{bmatrix} - & 1 & 0 & 0 \\ 0 & - & 0 & 0 \\ 1 & 1 & - & 0 \\ 1 & 1 & 0 & - \end{bmatrix}$$

8. Aggregate the dominance matrix $M_{aggregated\ dominance}$.

   The aggregated dominance matrix is calculated by an element-to-element product of the concordance dominance matrix and the discordance dominance matrix.

$$M_{aggregated\ dominance} = \begin{array}{cc} & \text{is dominated by} \\ \text{dominates} & \begin{bmatrix} - & 1 & 0 & 0 \\ 0 & - & 0 & 0 \\ 1 & 0 & - & 0 \\ 0 & 1 & 0 & - \end{bmatrix} \end{array}$$

9. Eliminate the dominated alternatives.

   In the aggregated dominance matrix, the element 1 in the column indicates that this alternative is dominated by other alternatives. In this example, it can be identified that A_1 is dominated by A_3, A_2 is dominated by A_1 and A_4. Thus, A_1 and A_2 are dominated alternatives and can be excluded by ELECTRE I.

It can be obtained that when weighting factors are evenly distributed among the seven criteria, A_1 and A_2 are dominated by A_3 and A_4. In other words, A_1 (Bombardier Challenger 300) and A_2 (Cessna Citation X) should be excluded from the candidates of business jets. But the outranking relationship between A_3 (Gulfstream G200) and A_4 (Hawker H4000) cannot be identified in the current set of weighting factors.

6.3.2 Typical Weighting Scenarios for ELECTRE I

Weighting factors play an important role in the decision analysis process. In this study, in order to better simulate DM's preference information, several typical weighting scenarios for the seven criteria are generated from eleven levels of experimental design. The weighting factors for the seven criteria are the combination of seven numbers from the set [0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1], with the constraint that the sum is one. Since the seven decision criteria need to be considered simultaneously in the decision analysis process, all the seven numbers are required to be bigger than zero. Thus, 84 sets of weighting factors are generated and attached in Table D.4 in Appendix D.3.

The weighing factors reflect the relative importance of the decision criteria. For instance, the first row in Table D.4 is [0.4 0.1 0.1 0.1 0.1 0.1 0.1]. This set of weighting factors indicates that C_1 (fuel consumption per seat kilometer) is the most important decision criterion, and the other six decision criteria have the same level of importance. The other 83 sets of weighting factors have similar explanations.

The evaluation results using ELECTRE I for the 84 sets of weighting factors are summarized in Table 6.7. It is observed that when the DM takes into account all the seven criteria, A_4 has the highest frequency to be the non-dominated alternative, and A_2 has the highest frequency to be the dominated alternative. Therefore, it can be concluded that for the scenario considered in this study, A_4 (Hawker H4000) should be recommended for the business aviation customer to purchase and A_2 (Cessna Citation X) should be excluded from the candidates of business jets.

Table 6.7: Evaluation Results for 84 Sets of Weighting Factors using ELECTRE I

	A_1	A_2	A_3	A_4
Non-dominated times	50	34	51	59
Dominated times	34	50	33	25
Non-dominated frequency	59.52%	40.48%	60.71%	70.24%
Dominated frequency	40.48%	59.52%	39.29%	29.76%

6.4 Uncertainty Assessment

In the business aircraft evaluation process, weighting factors and criteria values are the main input data utilized to solve the decision problem. It is observed that the weighting factors are often highly subjective considering the fact that they are elicited based on the DM's experience or intuition, while there are always uncertainties existing in the criteria values due to incomplete information. The inherent uncertainties and subjectivities of the input parameters have significant impacts on the final result of a decision making problem. Thus, it is critical to effectively address these uncertainties in the decision making process in order to get more accurate results. In this section, uncertainty assessment for weighting factors and criteria values is performed, following the new uncertainty assessment approach proposed in Chapter 4.

6.4.1 Uncertainty Characterization

As discussed previously in Section 4.1, uncertainties for weighting factors and criteria values are described by percentage uncertainties with different confidence levels first.

Suppose that the DM assigns 15% uncertainty to the weight of the first decision criterion (w_1) with 90% confidence level. In other words, the DM is 90 percent confident that w_1 would fall within the interval $[w_1(1-15\%), w_1(1+15\%)]$. The percentage untertainties and confidence levels of other weighting factors and criteria values have similar explanation. The weighting factors and criteria values with percentage uncertainties and confidence levels are summarized in Table 6.8.

Secondly, percentage uncertainties with different confidence levels are transferred into standard deviations using inverse error function, as described in Equation 4.4 and Equation 4.6 in Subsection 4.1.2.

When the weighting factors are evenly distributed among the seven decision criteria,

Table 6.8: Uncertainty Characterization for Weighting Factors and Criteria Values

	Weighting factors						
	w_1	w_2	w_3	w_4	w_5	w_6	w_7
Percentage Uncertainty	15%	10%	15%	10%	25%	30%	30%
Confidence Level	90%	95%	85%	90%	70%	80%	90%
	Criteria values						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Percentage Uncertainty	10%	5%	15%	10%	20%	20%	20%
Confidence Level	90%	90%	85%	95%	80%	90%	95%

the mean of weighting factors μ_W equals to normalized weighting factors. The standard deviation of weighting factors σ_W is shown as follows.

$$\mu_W = [0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.14290.1429\ ]^{\mathrm{T}}$$

$$\sigma_W = [0.0130\ 0.0073\ 0.0149\ 0087\ 0.0345\ 0.0335\ 0.0261]^{\mathrm{T}}$$

For instance, the standard deviation of w_1 with 15% uncertainty at 90% confidence level, is calculated by Equation 6.4 and Equation 6.5.

$$\begin{aligned} n_{w_1} &= \sqrt{2}erf^{-1}(\text{Confidence level}) \\ &= \sqrt{2}erf^{-1}(90\%) \\ &= 1.6449 \end{aligned} \tag{6.4}$$

$$\begin{aligned} \sigma_{w_1} &= \frac{\text{Relative error}(\%)\mu_{w_1}}{n_{w_1}} \\ &= \frac{(15\%)(0.1429)}{1.6449} \\ &= 0.0130 \end{aligned} \tag{6.5}$$

The similar calculation is carried out for other weighting factors and criteria values. The normalized decision matrix D can be taken as μ_D, and the standard deviation of the decision matrix is shown in σ_D.

$$\mu_D = \begin{bmatrix} 0.5178 & 0.4881 & 0.5423 & 0.5035 & 0.6149 & 0.4879 & 0.4175 \\ 0.4561 & 0.5341 & 0.5073 & 0.5145 & 0.3577 & 0.5257 & 0.2960 \\ 0.5480 & 0.4881 & 0.4288 & 0.4890 & 0.4707 & 0.4956 & 0.6225 \\ 0.4728 & 0.4881 & 0.5145 & 0.4926 & 0.5219 & 0.4899 & 0.5921 \end{bmatrix}$$

$$\sigma_D = \begin{bmatrix} 0.0191 & 0.0090 & 0.0393 & 0.0178 & 0.0490 & 0.0412 & 0.0296 \\ 0.0169 & 0.0099 & 0.0367 & 0.0182 & 0.0285 & 0.0444 & 0.0210 \\ 0.0203 & 0.0090 & 0.0310 & 0.0173 & 0.0375 & 0.0419 & 0.0441 \\ 0.0175 & 0.0090 & 0.0372 & 0.0175 & 0.0416 & 0.0414 & 0.0420 \end{bmatrix}$$

In this step, uncertainties in the weighting factors and criteria values, characterized by percentage uncertainties and confidence levels, are transferred into means and standard deviations. μ_D, μ_W, σ_D, and σ_W are the input for the error propagation calculation in the uncertainty analysis step.

6.4.2 Uncertainty Analysis

As noted in Section 4.2, Monte Carlo-based numerical error propagation technique is applied to perform uncertainty analysis when using ELECTRE I. 10,000 runs are performed from normal distribution with parameters μ_D, μ_W, σ_D, and σ_W. In this study, three scenarios are considered: uncertainty propagated from the weighting factors, criteria values, and both from the weighting factors and criteria values, as summarized in Table 6.9.

Table 6.9: Three Scenarios for Uncertainty Analysis

	Uncertainty incorporation	
Scenario	Weighting factors	Criteria values
1	✓	
2		✓
3	✓	✓

The probabilistic outranking relationships for each alternative in the three scenarios are presented in Table 6.10. It can be observed that in the three scenarios, with evenly distributed weighting factors among the seven decision criteria, A_4 (Hawker H4000) has the highest probability to be non-dominated, while A_2 (Cessna Citation X) has the highest probability to be dominated. The results of uncertainty analysis are consistent with the evaluation results of 84 sets of weighting factors presented in Table 6.7 in Subsection 6.3.2.

Table 6.10: The Probabilistic Outranking Relationships in Three Scenarios

	Alternatives			
	A_1	A_2	A_3	A_4
Scenario 1				
Non-dominated	48.84%	11.50%	89.22%	99.71%
Dominated	51.16%	88.50%	10.78%	0.29%
Scenario 2				
Non-dominated	67.79%	9.16%	64.93%	72.37%
Dominated	32.21%	90.84%	35.07%	27.63%
Scenario 3				
Non-dominated	67.20%	10.04%	63.98%	70.34%
Dominated	32.80%	89.96%	36.02%	29.66%

Besides, it also should be noted that in the three scenarios, the non-dominance or dominance status of A_2, A_3, and A_4 are preserved, while the dominance status of A_1 is not preserved in scenario 2 and scenario 3. The unstable status of A_1 can be attributed to its sensitivity to the weighting factors and criteria values. The sensitivity of the alternatives to the weighting factors and criteria values will be investigated in the following sensitivity analysis subsections.

Confidence Quantification of Sampling-based Error Propagation Technique

Since the numerical error propagation technique is sampling-based, a large number of samples are required in order to recreate the probability distributions of the input parameters. In our case, 10,000 Monte-Carlo simulation runs are performed in the uncertainty analysis process. However, with the same input parameters, the results of uncertainty analysis will not be the same because of the randomness of the sampling method. In this study, the degree of confidence of the uncertainty analysis results is quantified through confidence intervals. The nested simulation loop for the confidence quantification is shown in Figure 6.7.

Considering that the mean and standard deviation for the 10,000 Monte-Carlo sim-

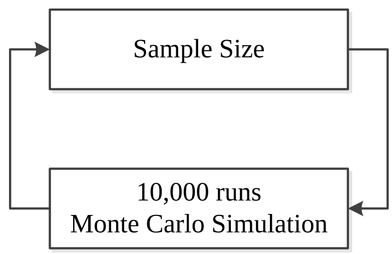


Figure 6.7: Nested Monte Carlo Simulation Loop for Confidence Quantification

ulation runs are unknown, we can suppose that the sample mean $\overline{x}$ follows t distribution with mean μ and standard deviation $s/\sqrt{n}$, where s is the estimated standard deviation, n is the sample size (62). The t distribution with sample size n has $n-1$ degree of freedom. The confidence interval is calculated by Equation 6.6.

$$[\overline{x} - t^* s/\sqrt{n}, \overline{x} + t^* s/\sqrt{n}] \tag{6.6}$$

where t^* is the upper $(1-\mathrm{CL})/2$ critical value for the t distribution with $n-1$ degree of freedom, CL is confidence level.

In this example, we take the sample size $n = 100$, $\mathrm{CL} = 95\%$, the 0.025 critical value for 99 degree of freedom is $t^* = 1.984$. The 95% confidence intervals for the probabilistic outranking relationship in the three scenarios are summarized in Table 6.11. For instance, for the non-dominance probability of A_1 in Scenario 1, the sample mean is 48.35%, the sample standard deviation is 0.5535%. The 95% confidence interval is calculated by Equation 6.7.

$$\begin{aligned}
[\overline{x} - t^* s/\sqrt{n}, \overline{x} + t^* s/\sqrt{n}] &= [0.4835 - 1.984 \times \ 0.005535/\sqrt{100}, \\
&\quad 0.4835 + 1.984 \times \ 0.005535/\sqrt{100}] \\
&= [48.24\%, 48.46\%]
\end{aligned} \tag{6.7}$$

The tight confidence intervals in Table 6.11 verify that the sampling-based error propagation technique can generate accurate results in the uncertainty analysis for the business aircraft evaluation.

Table 6.11: The 95% Confidence Intervals for the Probabilistic Outranking Relationship in Three Scenarios

	Alternatives			
	A_1	A_2	A_3	A_4
Scenario 1				
Non-dominated	[48.24%,48.46%]	[11.53%,11.68%]	[89.81%,89.93%]	[99.74%,99.76%]
Dominated	[51.54%,51.76%]	[88.32%,88.47%]	[10.07%,10.19%]	[0.24%,0.26%]
Scenario 2				
Non-dominated	[67.46%,67.65%]	[9.00%,9.13%]	[64.69%,64.88%]	[72.02%,72.21%]
Dominated	[32.35%,32.54%]	[90.87%,91.00%]	[35.12 %,35.31%]	[27.79%,27.98%]
Scenario 3				
Non-dominated	[67.07%,67.24%]	[9.68%,9.80%]	[63.65%,63.86%]	[70.47%,70.65%]
Dominated	[32.76%,32.93%]	[90.20%,90.32%]	[36.14%,36.35%]	[29.35%,29.53%]

6.4.3 Sensitivity Analysis

Local sensitivity analysis based on iterative binary search algorithm and global sensitivity analysis using partial rank correlation coefficients are conducted for the business aircraft evaluation problem in the following subsections, respectively.

Local Sensitivity Analysis Based on Iterative Binary Search Algorithm

As discussed in Section 4.3, local sensitivity analysis varies input variables one at a time to determine which variables have the greatest effect on the model output, while holding the others fixed at nominal values. In the business aircraft evaluation problem using ELECTRE I, with equally distributed weighting factors among the seven criteria, A_3 and A_4 are non-dominated alternatives, while A_1 and A_2 are dominated alternatives. The developed iterative binary search algorithm can answer the question: What is the minimum change in the weighting factors or criteria values so that the non-dominance or dominance status of an alternative can be altered?

Local Sensitivity Analysis for Weighting Factors

The absolute minimum change in the weighting factors which can alter the non-dominance or dominance status of alternatives are summarized in Table 6.12, where N/F (Non-Feasible) means that it is not mathematically feasible to alter the non-dominance or dominance status of alternatives through the change of the current parameter.

For the convenience of comparison, the relative minimum changes in the weighting factors which can alter the non-dominance or dominance status of alternatives are also presented in Table 6.13, where *Non.* represents non-dominated and *Dom.* represents dominated. The relative minimum changes are the absolute minimum changes scaled against the original values of weighting factors.

Table 6.12: Absolute Minimum Changes in Weighting Factors to Alter the Non-dominance or Dominance Status of Alternatives

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1 to Non.	-0.0396	N/F	0.0416	0.0716	0.0048	-0.0715	-0.0049
A_2 to Non.	-0.0715	0.0478	-0.0715	0.0716	-0.0715	0.0716	-0.0715
A_3 to Dom.	-0.0272	0.5814	0.0324	1.4440	0.1281	1.6962	-0.0632
A_4 to Dom.	0.0868	0.8808	-0.0550	1.9280	0.2535	1.1968	-0.0841

Table 6.13: Relative Minimum Changes in Weighting Factors to Alter the Non-dominance or Dominance Status of Alternatives

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1 to Non.	-27.67%	N/F	29.05%	50.06%	3.33%	-49.99%	-3.41%
A_2 to Non.	-49.99%	33.39%	-49.99%	50.06%	-49.99%	50.06%	-49.99%
A_3 to Dom.	-18.98%	406.80%	22.65%	1010.50%	89.59%	1186.95%	-44.20%
A_4 to Dom.	60.69%	616.37%	-38.47%	1349.15%	177.39%	837.45%	-58.81%

It can be seen from the first row in Table 6.13 that for dominated alternative A_1, it is not feasible to change the weighting factor of C_2 to switch A_1 into non-dominated alternative, while only around 3% increase in C_5 or around 3% decrease in C_7 can make A_1 become non-dominated alternative. Therefore, it can be concluded that A_1 is most robust against the weighting factor of C_2 and most sensitive to the weighting factors of C_5 and C_7.

Interactive Sensitivity Analysis for Weighting Factors

In this study, interactive sensitivity analysis for the weighting factors is developed with the purpose of providing the DM more vivid decision aiding, as shown in Figure 6.8, where the green bar represents that the alternative is non-dominated. The DM can simply move the slide bar of the weighting factor, and the change of the non-dominance or dominance status of the four alternatives is displayed simultaneously. The main idea of the interactive sensitivity analysis of weighting factors is to vary the weighting of one criterion from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting.

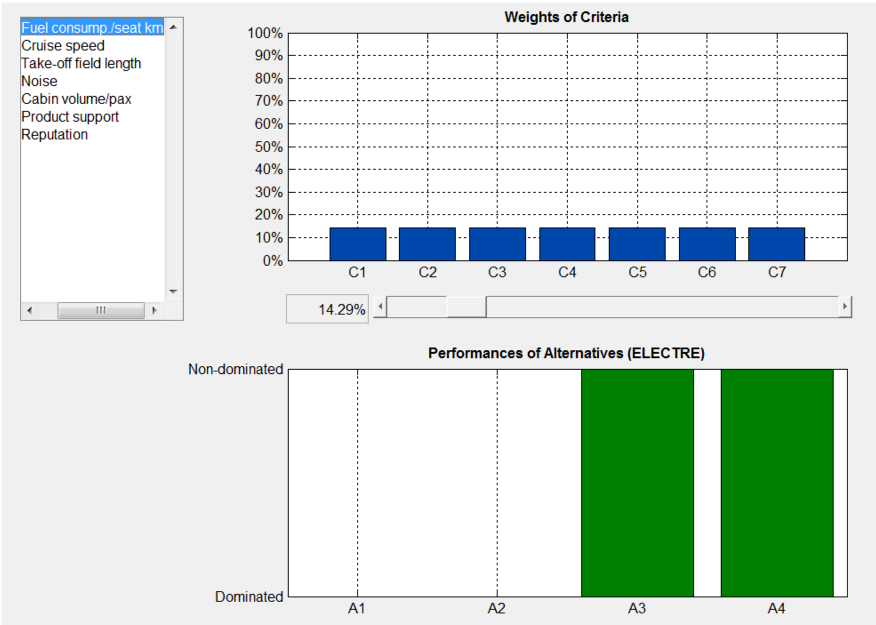


Figure 6.8: Interactive Sensitivity Analysis for Weighting Factors

The interactive weighting plot for C_1 is presented in Figure 6.9, the plots for the other six criteria (C_2 to C_7) are attached in Appendix C.3. The four alternatives are marked with different colors. The count of the vertical line stands for the change frequency of non-dominance or dominance status for one alternative.

For instance, in Figure 6.9, the purple line represents A_4, one purple vertical line tell us that when varying the weighting of C_1 from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting, A_4 changes one time from non-dominated to dominated alternative. Similarly, it can be observed that A_1 changes five times, A_2 and A_3 change one time, respectively.

The frequency of status change for the four alternatives, when varying the weighting

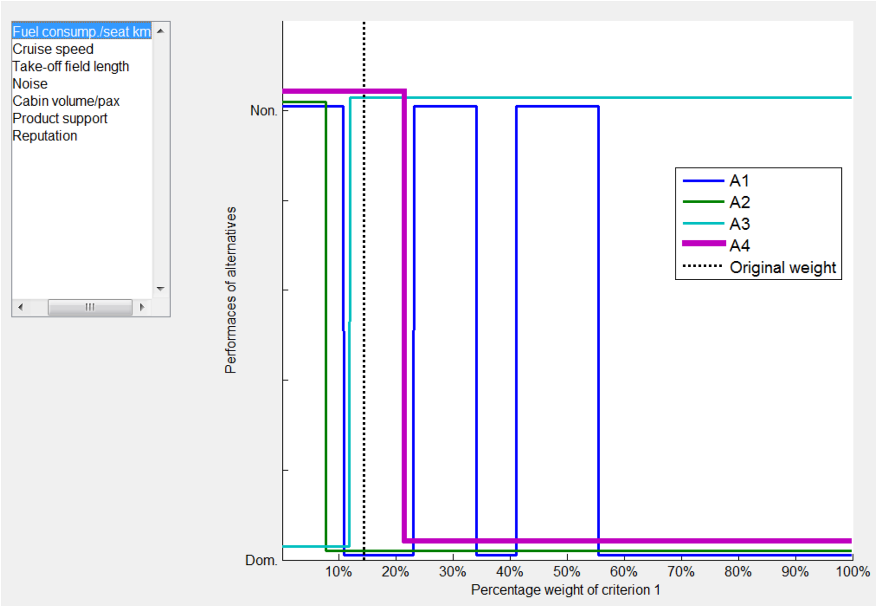


Figure 6.9: Interactive Weighting Plot for Criterion 1

factors of the seven decision criteria from 0 to 100% individually, are summarized in Table 6.14. The row sum represents that for one alternative, how many times the status of this alternative has been changed, when varying the weighting factors of the seven decision criteria from 0 to 100% individually. The column sum represents that for one criterion, how many times the non-dominance or dominance status of the four alternatives have been changed, when varying the weighting of this criterion from 0 to 100%.

Table 6.14: Frequency of Status Change for Alternatives in Interactive Weighting Plots

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Row sum
A_1	5	0	2	2	1	1	1	12
A_2	1	1	1	1	1	1	1	7
A_3	1	3	1	1	1	1	1	9
A_4	1	1	2	1	1	1	2	9
Column sum	8	5	6	5	4	4	5	

In Table 6.14, the biggest column sum of C_1 shows that C_1 has the highest frequency to change the non-dominance or dominance status of the four alternatives, when varying the weighting of this criterion from 0 to 100%. The biggest row sum of A_1 shows that A_1 has the highest frequency of changing the non-dominance or dominance status, when varying the weighting factors of the seven decision criteria from 0 to 100%, individually. In other words, among the four alternatives, A_1 is most sensitive to the weighting factors of the seven decision criteria. The sensitivity of A_1 to the weighting factors is consistent with the results shown in Table 6.12 and Table 6.13.

Furthermore, it is important to note that Table 6.12, Table 6.13, and Table 6.14 address different aspects of local sensitivity analysis for the weighting factors. Table 6.12 and Table 6.13 show the minimum changes in the weighting factors when the non-dominance or dominance status of alternatives is altered around the region of the nominal values of the weighting factors, which are located in the vicinity of the dot-dashed line in the interactive weighting plots. Table 6.14 summarizes the total frequency for the non-dominance or dominance status change of alternatives when varying the weighting of one criterion from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting.

Local Sensitivity Analysis for Criteria Values

Local sensitivity analysis for criteria values investigates how to change the criteria values so that the non-dominance or dominance status of alternatives can be altered. The developed iterative binary search algorithm can provide the mathematically feasible change of the criteria values to alter the non-dominance or dominance status of alternatives. However, for the business aircraft evaluation problem, mathematical feasibility does not necessarily guarantee physical feasibility. For instance, when the value of C_2 (high-speed cruise speed) is changed, it should be less than its maximum operating speed. The physical constraints of the decision criteria in the business aircraft evaluation problem are summarized in Table 6.15. Any change which violates these constraints is physically non-feasible.

In Table 6.15, M_{MO} represents Maximum operating Mach number. According to BCA (2), the M_{MO} for the four business jets are 1016 km/h (0.83 Mach), 1126 km/h (0.92 Mach), 1040 km/h (0.85 Mach), and 1028 km/h (0.84 Mach), respectively. The constraint for C_5 is calculated by $42.5/8 = 5.3125$, which is based on the maximum cabin volume per passenger for the medium jets, as shown in Table 6.1. The constraint for the

Table 6.15: Physical Constraints of the Decision Criteria for Business Aircraft

Decision Criteria	Constraints
C_1: Fuel consumption per seat kilometer (kg/pax/km)	-
C_2: High-speed cruise speed (km/h)	$\leq M_{MO}$
C_3: Take-off field length (m)	[1300, 1900]
C_4: Noise (EPNdB)	[80, 90]
C_5: Cabin volume per passenger (m^3/pax)	$\leq$ 5.3125
C_6: Product support level	[1,10]
C_7: Manufacturer's reputation	[1,99]

product support level is based on the overall average scores obtained via the aviation international news 2010 product survey, as shown in Figure 6.3. The constraint for manufacturer's reputation is based on the aviation week's 16th annual top-performing companies study for 2010, as summarized in Table 6.4.

The absolute minimum changes in the criteria values which can alter the non-dominance or dominance status of alternatives are summarized in Table 6.16, where N/F (Non-Feasible) represents that it is not mathematically feasible to alter the non-dominance or dominance status of alternatives through the change of the current parameter, and PN/F (Physically Non-Feasible) represents that the changed parameter violates its physical constraints.

For the convenience of comparison, the relative minimum changes in the criteria values which can alter the non-dominance or dominance status of alternatives are summarized in Table 6.17. The relative minimum changes are the absolute minimum changes scaled against the original criteria values of the alternatives.

The first four-rows in Table 6.17 show the minimum changes in the criteria values of A_1 so that the non-dominance or dominance status of the four alternatives can be altered. It can be seen that it is not feasible to change any criteria value of A_1 in order to alter the dominance status of A_2.

Similarly, it can be observed from the second four-rows in Table 6.17 that it is not feasible to change any criteria value of A_2 so that the non-dominance or dominance status of A_1, A_2, and A_4 can be altered. The third four-rows show that it is not feasible to change any criteria value of A_3 in order to alter the dominance status of A_2. The

Table 6.16: Absolute Minimum Changes in Criteria Values to Alter the Non-dominance or Dominance Status of Alternatives

Cri.values changed	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Alt. status changed
A_1	N/F	0.01	N/F	N/F	N/F	0.13	N/F	A_1
A_1	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_2
A_1	-0.11	PN/F	N/F	PN/F	N/F	PN/F	N/F	A_3
A_1	-0.08	N/F	PN/F	N/F	1.13	N/F	13.5	A_4
A_2	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_1
A_2	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_2
A_2	N/F	N/F	PN/F	PN/F	N/F	N/F	53.26	A_3
A_2	N/F	N/F	N/F	N/F	PN/F	N/F	PN/F	A_4
A_3	N/F	PN/F	-122.19	PN/F	N/F	2.19	N/F	A_1
A_3	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_2
A_3	0.01	-175.08	PN/F	PN/F	-0.33	-1.63	-13.99	A_3
A_3	-0.02	PN/F	N/F	N/F	N/F	1.93	N/F	A_4
A_4	-0.02	PN/F	N/F	N/F	N/F	N/F	N/F	A_1
A_4	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_2
A_4	-0.01	PN/F	-61.51	PN/F	0.42	1.85	19.61	A_3
A_4	0.03	-192.24	106.68	PN/F	-0.84	-1.62	-10.45	A_4

fourth four-rows show that it is not feasible to change any criteria value of A_4 so that the dominance status of A_2 can be modified.

The whole Table 6.17 shows that the criterion value C_2 of A_1 is most sensitive to the dominance status of A_1, and the criterion value C_4 is most robust against the change of the non-dominance or dominance status of the four alternatives.

Summary of Local Sensitivity Analysis for Weighting Factors and Criteria Values

According to the results of local sensitivity analysis for the weighting factors and criteria values shown in Table 6.13 and Table 6.17, we can summarize that in this business aircraft evaluation problem, A_1 is most sensitive to the weighting factors of C_5 and C_7 and the criteria value of C_2, and the criterion value C_4 is most robust against the change of the non-dominance or dominance status of the four alternatives. The sensitivity of A_1 explains its unstable status shown in Table 6.10.

Attention should be paid that these minimum changes in the weighting factors and criteria values, shown in Table 6.13 and Table 6.17, are obtained using local sensitivity

Table 6.17: Relative Minimum Changes in Criteria Values to Alter the Non-dominance or Dominance Status of Alternatives

Cri.values changed	C_1	C_2	C_3	C_4	C_5	C_6	C_7	Alt. status changed
A_1	N/F	0.01%	N/F	N/F	N/F	1.58%	N/F	A_1
A_1	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_2
A_1	-42.86%	PN/F	N/F	PN/F	N/F	PN/F	N/F	A_3
A_1	-29.88%	N/F	-30.81%	N/F	27.70%	N/F	24.55%	A_4
A_2	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_1
A_2	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_2
A_2	N/F	N/F	-41.10%	PN/F	N/F	N/F	136.56%	A_3
A_2	N/F	N/F	N/F	N/F	PN/F	N/F	PN/F	A_4
A_3	N/F	PN/F	-6.60%	PN/F	N/F	28.22%	N/F	A_1
A_3	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_2
A_3	2.78%	-20.13%	PN/F	PN/F	-10.39%	-21.03%	-17.06%	A_3
A_3	-8.20%	PN/F	N/F	N/F	N/F	24.83%	N/F	A_4
A_4	-4.12%	PN/F	N/F	N/F	N/F	N/F	N/F	A_1
A_4	N/F	N/F	N/F	N/F	N/F	N/F	N/F	A_2
A_4	-2.86%	PN/F	-3.99%	PN/F	11.98%	24.15%	25.15%	A_3
A_4	10.06%	-22.10%	6.91%	PN/F	-24.17%	-21.15%	-13.40%	A_4

analysis. In other words, only one variable is varied at a time around its nominal value and the interactions among the input variables may not be captured. The simultaneous variations of all variables and the effects of the interactions among the input variables are investigated in global sensitivity analysis in the next subsection.

Global Sensitivity Analysis Using Partial Rank Correlation Coefficients

In contrast to local sensitivity analysis, global sensitivity analysis allows the variations of all variables over the full range at the same time. In this subsection, global sensitivity analysis using partial rank correlation coefficients for the business aircraft evaluation problem is presented, according to the proposed approach in Section 4.4.

Step 1: Define Probability Distributions for Input Variables

In the business aircraft evaluation problem using ELECTRE I, input variables are seven decision criteria and weighting factors. The outputs are the outranking relationships for the four alternatives. Since there is no sufficient data to construct their probability distribution functions, uniform distribution is chosen for the fourteen input variables.

For the seven decision criteria, the physical constraints shown in Table 6.15 serve as the minimum and maximum values, where the range of C_1 and the minimum value of C_5 are given by an expert. The weighting factors range from 0.05 to 0.85 in order to take all seven criteria into consideration. The probability distributions for the fourteen input variables are summarized in Table 6.18.

Table 6.18: Probability Distributions for Input Variables

Input variables	Min	Max	Distribution
C_1	0.2	0.4	Uniform
C_2	850	1016	Uniform
C_3	1300	1900	Uniform
C_4	80	90	Uniform
C_5	2	5.3125	Uniform
C_6	1	10	Uniform
C_7	1	99	Uniform
$W_i, i = 1, .., 7$	0.05	0.85	Uniform

Step 2: Perform Latin Hypercube Sampling

The efficient LHS enables to vary all variables at the same time with low computational cost in global sensitivity analysis. In the business aircraft evaluation problem using ELECTRE I, 1000 LHS runs are carried out with the probability functions defined for the fourteen input variables in Step 1. The minimum value of sample size for LHS is $\frac{3}{4}k$, where k is the number of input parameters that are varied (11). In this example, $k = 14$, thus, 1000 runs of LHS is adequate for the calculation of partial rank correlation coefficients.

For each combination of the sampled values of the decision criteria and weighting factors, ELECTRE I is utilized to calculate the overall performances of the alternatives.

Step 3: Rank Transformation for both Input Variables and MCDA Output

In this step, the fourteen input variables and ELECTRE I output are transformed into ranks. Since ELECTRE I output is the outranking relationship of alternatives instead of scoring, the rank transformation is performed as described in Section 4.4. At first, the outrank set is assigned scores as follows: the non-dominated alternatives are assigned score 1, while the dominated alternatives are assigned score 0. Next, the outrank set with scores is transformed into ranks as the scoring MCDA methods, in a similar way but with tied ranks.

For example, in the business aircraft evaluation problem with equal weighting factors, A_3 and A_4 are non-dominated alternatives, while A_1 and A_2 are dominated alternatives. Thus, in the first step, A_3 and A_4 are assigned score 1, while A_1 and A_2 are assigned score 0. Next, the assigned score vector [0 0 1 1] is transformed into ranks. Counting from smallest to largest, the two 0 rank first and second, the average rank is $(1+2)/2 = 1.5$. The two 1 rank third and fourth, their average rank is $(3+4)/2 = 3.5$. Thus, the transformed ranks of the outrank set in ELECTRE I is [1.5 1.5 3.5 3.5].

Step 4: Calculate Partial Rank Correlation Coefficients

With the rank-transformed data, partial rank correlation coefficients can be calculated. The Tornado plots of partial rank correlation coefficients for the four alternatives are presented in Figure 6.10, where the corresponding p-values for the partial rank correlation coefficients are next to the bars.

Step 5: Conduct Statistical Significance Test

In this study, p-values are computed to assess the statistical significance of partial rank correlation coefficients, as shown in Figure 6.10. A lower p-value provides stronger evidence to reject the null hypothesis H_0 that there is no partial correlation, in favor of the alternative hypothesis H_1 that there is nonzero partial correlation between the rank transformed input variables and ELECTRE I output.

Step 6: Results Interpretation

Partial rank correlation coefficients should be interpreted together with statistical significance test. In this example, p-values less than 0.05 indicate that the partial rank correlation coefficients are statistically significant.

It is observed from Figure 6.10 that in the business aircraft evaluation problem using ELECTRE I, for the two non-dominated alternatives A_3 and A_4, input variable C_7 shows the strongest statistically significant correlations with the overall performances of the four alternatives, while for the two dominated alternatives A_1 and A_2, input variable C_5 shows the strongest statistically significant correlations with the overall performances of the four alternatives. Moreover, three input variables: C_5, C_6, and C_7, have the top three statistically significant correlations with the overall performances of the four alternatives.

The magnitude of partial rank correlation coefficients in global sensitivity analysis represents the relative importance of the influence of input variables on model outputs. Therefore, it is concluded that C_7 is most important for the performance of the non-dominated alternatives, C_5 is most important for the performance of the dominated alternatives, and C_5, C_6, and C_7, are most important in contributions to the overall performances of the four alternatives.

It should be kept in mind that there are two components in global sensitivity analysis: the range of input variable and the sensitivity coefficient of the output to this input variable (61). An input variable is identified as important in global sensitivity analysis if it has a wide range and large sensitivity coefficient. In our case, C_7 is detected as the most important input variable for the performance of the non-dominated alternatives may be contributed by its wide range (1-99).

It is interesting to note that the three most important variables: C_5, C_6, and C_7, based on the partial rank correlation coefficients in global sensitivity analysis, are the three additional soft decision criteria in the business aircraft evaluation problem. This proves that when evaluating the business aircraft, in addition to the technical *hard* criteria, it is also crucial to assess the additional soft criteria. The aggregation of the technical *hard* criteria and the additional soft criteria is the unique advantage of the MCDA methods.

Evaluation of Statistical Power of Partial Rank Correlation Coefficients

It is noted that when performing the global sensitivity analysis for ELECTRE I, the magnitudes of partial rank correlation coefficients are relative small, this may be attributed to the rank transformation approach performed in Step 3, and too many tied ranks reduce the statistical power of partial rank correlation coefficients. Thus, in order to assess the statistical power of partial rank correlation coefficients in the decision analysis process, one popular scoring method, TOPSIS, is also utilized to solve the business aircraft evaluation problem.

With the same input variables, the seven decision criteria shown in decision matrix

D and the weighting factors shown in W are repeated here for the convenience of calculation.

$$D = \begin{bmatrix} 0.2396 & 870 & 1466 & 84.2333 & 4.0500 & 7.63 & 55 \\ 0.2720 & 952 & 1567 & 82.4333 & 2.3556 & 8.22 & 39 \\ 0.2264 & 870 & 1854 & 86.7333 & 3.1000 & 7.75 & 82 \\ 0.2624 & 870 & 1545 & 86.1000 & 3.4375 & 7.66 & 78 \end{bmatrix}$$

$$W = [0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.1429\ 0.14290.1429\ ]^{\mathrm{T}}$$

The ranking of the four alternatives using TOPSIS are $[A_4\ A_3\ A_1\ A_2]$. The results are consistent with the evaluation results using ELECTRE I that A_3 and A_4 are non-dominated alternatives, while A_1 and A_2 are dominated alternatives.

The partial rank correlation coefficients for the four alternatives, when TOPSIS is utilized to solve the business aircraft evaluation problem, are presented in Figure 6.11, where the corresponding p-values for the partial rank correlation coefficients are next to the bars.

It is observed from Figure 6.11 that the three input variables: C_5, C_6, and C_7, have the top three statistically significant correlations with the overall performances of the four alternatives. This observation is consistent with when ELECTRE I is utilized to solve the business aircraft evaluation problem. Furthermore, the magnitudes of the partial rank correlation coefficients between the input variables and TOPSIS scores are bigger, which proves the statistical power of partial rank correlation coefficients in the decision analysis process.

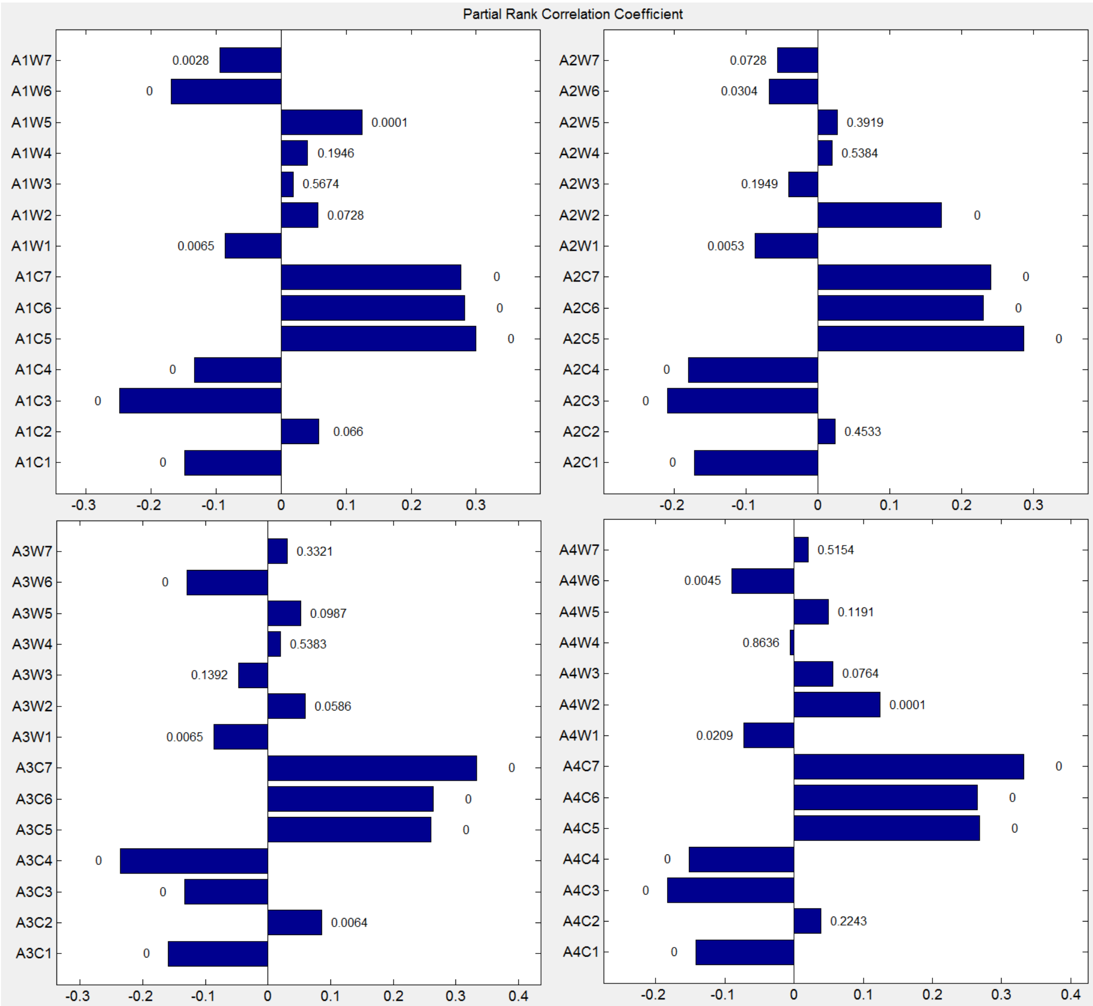


Figure 6.10: Tornado Plots of Partial Rank Correlation Coefficients for the Four Alternatives using ELECTRE I, with Corresponding p-values

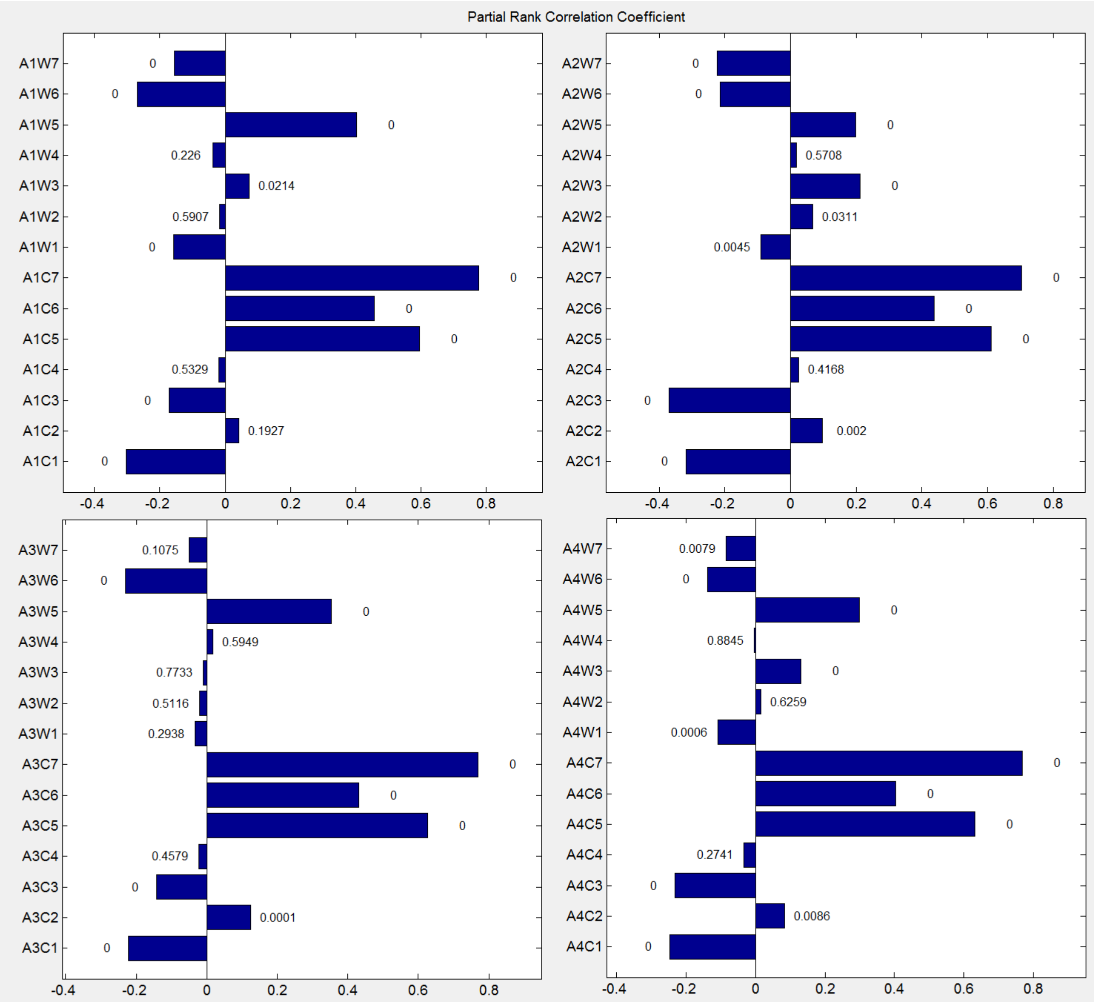


Figure 6.11: Tornado Plots of Partial Rank Correlation Coefficients for the Four Alternatives using TOPSIS, with Corresponding p-values

6.5 Discussion

The application of the most appropriate MCDA techniques in aircraft evaluation problems was presented in this chapter. A general framework was implemented following the three steps: definition of the decision making problem, selection of the most appropriate MCDA method, and uncertainty assessment in the decision analysis process. For the scenario considered in this study, A_4 (Hawker H4000) could be recommended for the business aviation customer to purchase, and A_2 (Cessna Citation X) should be excluded from the candidates of business jets.

In this section, the quantification of soft criteria is discussed first, followed by the advantages and disadvantages of local and global sensitivity analysis in the business aircraft evaluation problem.

Soft Criteria Quantification

The additional soft criteria are considered to be the decisive factors in the business aircraft evaluation problem. The quantification of the additional soft criteria was presented in Subsection 6.1.2.

Passenger comfort level was quantified by cabin volume per passenger. However, there are several other factors influencing passenger comfort, for instance, available seats and tables, bathroom facilities, passenger cabin electronics such as Internet, telephone, fax, reading lights, stereo sound systems, in-flight access to baggage, and in-flight food service. However, there is no available reliable data to quantify these factors. Thus, they are not included in the quantification of passenger comfort level. Further research is needed to quantify those factors for passenger comfort level.

Local Versus Global Sensitivity Analysis

As discussed in Section 4.3 and Section 4.4, local sensitivity analysis varies one input variable at a time, while global sensitivity analysis varies all variables simultaneously. Local sensitivity analysis has the advantages that the computation is efficient and it can provide the sensitivity of one individual variable on model output over a small region around the nominal values of input variables. However, when the model is nonlinear, or when several invariables are varied at the same time, local sensitivity analysis may not provide meaningful results. Global sensitivity analysis allows the variations of all input variables over their full range and can capture the effects of interactions among the input variables on the model outputs, but with higher computational costs.

In the business aircraft evaluation problem, in order to obtain an initial understanding of the sensitivity of one individual variable on the MCDA outputs over the region around the nominal values of input variables, local sensitivity analysis based on iterative binary search algorithm was conducted first. The results of local sensitivity analysis for the weighting factors and criteria values were summarized in Table 6.13 and Table 6.17, respectively.

In order to capture the effects of interactions among the weighting factors and criteria values on the MCDA outputs, global sensitivity analysis using partial rank correlation coefficients were also performed. The results of global sensitivity analysis were presented in Figure 6.10.

According to Table 6.13 and Table 6.17, the relative minimum changes of the input variables (weighting factors and criteria values) to alter the non-dominance or dominance status of the alternatives are ranked in ascending order. The top eight sensitive input variables identified by local sensitivity analysis are shown in Table 6.19, where the relative minimum changes are all less than 15%.

For the purpose of comparison, the values of partial rank correlation coefficients for the input variables are ranked in descending order, according to Figure 6.10. The top eight important input variables with statistical significance identified by global sensitivity analysis are also shown in Table 6.19, where the partial rank correlation coefficients are all bigger than 0.15.

As shown in Table 6.19, sensitivity rankings of the input variables identified by local sensitivity analysis and global sensitivity analysis are different. One reason would be that in local sensitivity analysis, the input variables are varied one at a time and the interactions among the input variables may not be captured.

However, this does not mean that the results of local sensitivity analysis are erroneous, because there are two distinct ways that the models are sensitive to the input variables (34): (1) small changes in the input variables result in significant changes in the model output, and (2) the variation of the input variables contributes substantially to the variation of the model output. The former input variables are called *sensitive*, and the latter input variables are called *important*. An *important* variable is always

Table 6.19: Comparison of Sensitivity Rankings for Input Variables Identified by Local and Global Sensitivity Analysis

Sensitivity rankings of input variables	Local sensitivity analysis	Global sensitivity analysis
1st	C_2	C_7
2nd	C_6	C_5
3rd	C_1	C_6
4th	W_5	C_3
5th	W_7	C_4
6th	C_3	C_1
7th	C_5	W_2
8th	C_7	W_6

sensitive because the variation of the variable will not appear in the model output unless the model is sensitive to this variable. However, a *sensitive* variable may not be *important* because the variable will have no influence on the variation of the model output if the value of the variable is known precisely (34).

The top four *important* input variables (C_7, C_5, C_6, and C_3) and the sixth *important* input variable (C_1), are recognized as *sensitive* by local sensitivity analysis, although the ranking orders are different. The fifth, seventh, and eighth *important* input variables (C_4, W_2, and W_6) are not recognized as *sensitive* by local sensitivity analysis, which can be attributed to the reason that they are insensitive by themselves, however, when interacted with other input variables, their variations contribute substantially to the variation of the MCDA output.

For the same reason, the first, fourth, and fifth *sensitive* input variables (C_2, W_5, and W_7), are not identified as *important* by global sensitivity analysis, because they are sensitive by themselves, however, when interacted with other input variables, their variations do not contribute greatly to the variation of the MCDA output.

In summary, we take the perspective that local sensitivity analysis and global sensitivity analysis investigate model behaviors in different domains of input variables (86), and global sensitivity analysis should not precede local sensitivity analysis (33). A complete understanding of the sensitivity of input variables on model outputs can be provided by performing both types of sensitivity analysis.

7

Conclusions

The goal of this research was to fill the gap between the MCDA theory and their practice in aerospace industry, by investigating the approaches how existing MCDA techniques could be improved to better solve decision making problems, and how the improved MCDA techniques could be implemented in aircraft design and evaluation decision analysis processes.

An advanced approach to effectively select the most appropriate MCDA method for a given problem was presented, and a new approach for uncertainty assessment in the decision analysis process was proposed, respectively. The first proof of concept was the implementation of an improved MCDA method with uncertainty assessment in aircraft conceptual design. The second proof of concept was the application of an appropriate MCDA technique with uncertainty assessment in business aircraft evaluation.

7.1 Research Questions Answered

Question 1: *How to select the most appropriate MCDA method for the decision making problem under consideration?*

There are several MCDA techniques available to solve decision making problems, where different methods have different underlying assumptions, information requirements, and decision rules that are designed for solving a certain class of decision making problems. Thus, it is important to select the most appropriate MCDA method for a given problem.

An advanced approach to effectively select the most appropriate MCDA method for a given problem was presented and an intelligent multi-criteria decision support system was developed. Twelve evaluation criteria were proposed to assess sixteen widely used MCDA methods. The match between the MCDA methods and a given problem was quantified by an appropriateness index, as proposed by **Hypothesis 1**. The MCDA method which has the highest appropriateness index would be recommended as the most appropriate method to solve the given problem.

Question 2: *How to capture and assess the uncertainties propagated in the decision analysis process when solving decision making problems?*

When using the MCDA techniques to solve decision making problems, weighting factors and decision criteria are the main input data. The weighting factors are often highly subjective considering the fact that they are elicited based on the DM's experience or intuition, while there are always uncertainties existing in the decision criteria due to incomplete information or limited knowledge. The inherent uncertainties of the input data in the decision analysis process have crucial impacts on the final solution for a decision making problem.

Hypothesis 2 proposed that statistical techniques are capable of effectively dealing with the uncertainties propagated in the decision analysis process. A new approach for uncertainty assessment was proposed. This approach consists of four steps: uncertainty characterization by percentage uncertainty with confidence level, uncertainty analysis using error propagation techniques, local sensitivity analysis based on iterative binary search algorithm, and global sensitivity analysis using partial rank correlation coefficients. The proposed approach was implemented and an uncertainty assessment module was integrated into the developed multi-criteria decision support system.

Question 3: *How to implement the improved MCDA techniques in aircraft design and aircraft evaluation decision making processes?*

As proposed by **Hypothesis 3**, a new optimization framework incorporating MCDA techniques in aircraft conceptual design process was established. The developed intelligent multi-criteria decision support system was used to select an appropriate MCDA method. It was demonstrated that the chosen MCDA method with improvement (ITOPSIS) provides a better objective function for the optimization than the traditional weighted sum (SAW) method. Furthermore, considering that the inherent uncertainties and subjectivities of the weighting factors have crucial impacts on the design solution, surrogate models for the multiple design criteria in terms of the weighting factors are constructed. Results show that the constructed surrogate models can enable efficient uncertainty assessment for the weighting factors.

In the application of the MCDA techniques in business aircraft evaluation process, the selection of the most appropriate MCDA method is conducted through the developed intelligent multi-criteria decision support system. In addition to the technical *hard* criteria, the soft criteria are considered to be the decisive factors in decision analysis process. In the business aircraft evaluation process, three soft criteria: passenger comfort level, product support level, and manufacturer's reputation, are considered and quantified. The synergy of the technical *hard* criteria and the additional soft criteria is the unique advantage of the MCDA methods.

7.2 Summary of Scientific Contributions

The main scientific contributions of this research are summarized as follows.

1. An advanced approach to effectively select the most appropriate MCDA method for a given problem is presented. This MCDA method selection approach is implemented and an intelligent multi-criteria decision support system is developed.

2. New uncertainty assessment approach in the decision analysis process is proposed, consisting of uncertainty characterization, uncertainty analysis, local sensitivity analysis, and global sensitivity analysis. The proposed uncertainty assessment approach is capable of filling the gap of propagating uncertainties in an assessment chain. When aggregating the results from different analysis tools, different levels of uncertainty associated with the different tools can be effectively captured by percentage uncertainties and confidence levels. Moreover, the step by step approach to perform global sensitivity analysis using partial rank correlation coefficients can be extended to investigate statistical relationships between variables in complex analysis problems.

3. A three-step framework for solving decision making problems is implemented: definition of a decision making problem, selection of the most appropriate MCDA method for the given problem, and uncertainty assessment in the decision analysis process. This framework provides a general guideline on how to structure and solve any given decision making problems.

7.3 Recommendations

This section discusses the recommendations for future work. Regarding the proposed approach for uncertainty assessment, global sensitivity analysis was based on partial rank correlation coefficients, with the assumption that the relationships between input variables and model outputs are monotonic. If non-monotonicities exist, variance decomposition analysis should be used to perform global sensitivity analysis.

In the established optimization framework incorporating MCDA techniques in aircraft conceptual design process, gradient-based methods were used. The reason is that the focus of this research has been on developing the framework of incorporating MCDA techniques in aircraft design process, particularly on exploring the feasibility and assessing the added values, not on the optimization itself. Genetic algorithms or hybrid optimizers combining genetic algorithms and gradient-based methods could be also investigated in the future.

Soft criteria are considered to be the decisive factors in decision analysis process. In the business aircraft evaluation process, three soft criteria were considered and quantified: passenger comfort level, product support level, and manufacturer's reputation. Further research could be done about the consideration and quantification of other soft criteria, such as pilot comfort.

The MCDA techniques with uncertainty assessment were implemented in aircraft design and aircraft evaluation decision making processes, respectively. The application of the MCDA techniques with uncertainty assessment could be extended into the assessment of the whole air transportation systems, for balancing social, economic, ecological and technical etc. constraints.

References

[1] European Aeronautics: a vision for 2020. Meeting society's needs and winning global leadership (2001). Luxembourg: Office for Official Publications of the European Communities, 2001. 1

[2] Business and Commercial Aviation Purchase Planning Handbook, May 2011. x, 118, 123, 142

[3] S. Ahlroth, M. Nilsson, G. Finnveden, O. Hjelm, and E. Hochschorner. Weighting and valuation in selected environmental system analysis tools suggestions for further developments. *Journal of Cleaner Production*, 19:145–156, 2011. 169

[4] J. Anselmo and A. Velocci. Aviation Week's 16th Annual Top-Performing Companies Study, June 2011. xiv, 121, 122

[5] N. Antoine, I. Kroo, K. Willcox, and G. Barter. A framework for aircraft conceptual design and environmental performance studies. In *10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, Albany, New York*, 30 August - 1 September 2004. 2, 83

[6] O. Bandte. *A Probabilistic Multi-Criteria Decision Making Technique for Conceptual and Preliminary Aerospace System Design.* PhD thesis, School of Aerospace Engineering, Georgia Institute of Technology, 2000. 3

[7] V. Belton and T. J. Stewart. *Multiple Criteria Decision Analysis - An Integrated Approach.* Kluwer Academic Publishers, 2002. 2, 8, 60

[8] R. Benayoun, B. Roy, and N. Sussman. Manual de reference du programme electre. *Psychoemtrika*, 38:337–369, 1973. 12

[9] C. Berger, R. Blauth, D. Boger, C. Bolster, G. Burchill, W. DuMouchel, F. Pouliot, R. Richter, A. Rubinoff, D. Shen, M. Timko, and D. Walden. Kanos methods for understanding customer-defined quality. *Center for Quality Management Journal*, Fall:3–35, 1993. xi, 169

[10] P. R. Bevington. *Data Reduction and Error Analysis for the Physical Sciences.* McGraw-Hill company, 1969. 48, 49

[11] S. Blower and H. Dowlatabadi. Uncertainty analysis of complex models of diseases transmission. *International Statistical Review*, 62:229–243, 1994. 60, 64, 65, 66, 146

[12] D. Boehnke, B. Nagel, and V. Gollnick. An approach to multi-fidelity in distributed design environments. In *IEEE Aerospace Conference,Big Sky, USA*, 2011. 72

[13] Bombardier. Bombardier Business Aircraft Market Forest 2011-2030, June 2011. xiv, 116

[14] D. Bouyssou. Some remarks on the notion of compensation in mcdm. *European Journal of Operational Research*, 26:150–160, 1985. 7

[15] J. Brans and P. Vincke. A preference ranking organization method: The promethee method for mcdm. *Management Science*, 31:647–656, 1985. 23

[16] J. Brans, P. Vincke, and B. Mareschal. How to select and how to rank projects: The promethee method. *European Journal of Operational Research*, 24:228–238, 1986. ix, 23, 24

[17] H. C. Calpine and A. Golding. Some properties of pareto-optimal choices in decision problems. *OMEGA*, 4:141–147, 1976. 9, 11

[18] G. Chen, Y. Han, H.-G. Nuesser, and D. Wilken. A method of evaluating civil aircraft market adequacy. In *DGLR-Workshop Aircraft Evaluation*, 1998. 3

[19] W. Chen. On the problem and elimination of rank reversal in the application of topsis method. *Operations Research and Management Science*, 14:50–55, 2005. 82

[20] E. Choo, B. Schoner, and W. Wedley. Interpretation of criteria weights in multi-criteria decision making. *Computers and Industrial Engineering Journal*, 37:527–541, 1999. 7

[21] Y. Collette and P. Siarry. *Multiobjective Optimization: Principles and Case Studies*. Springer, 2003. 12, 16

[22] J. Davis, W. Hands, and U. Maki. *Handbook of Economic Methodology*. Edward Elar, 1997. 21

[23] K. Deb. *Multi-Objective Optimization using Evolutionary Algorithms*. Wiley, 2001. 83, 84

[24] K. Deb, A. Sinha, P. Korhonen, and J.Wallenius. An interactive evolutionary multi-objective optimization method based on progressively approximated value functions. *IEEE Transactions on Evolutionary Computation*, 14:723–739, 2010. 83

[25] H. Dehling. Daniel bernoulli and the st. petersburg paradox. *Vierde serie Deel*, 15:223–227, 1997. 21, 22

[26] G. A. Dirks and F. Meller. Multidiciplinary design optimization - enhanced methodology for aircraft and technology evaluation. In *8th AIAA/USAF/ISSMO Symposium on Multidiciplinary Analysis and Optimization, Long Beach, CA*, 2000. 1, 3

[27] W. Edwards. How to use multiattribute utility measurement for social decision making. *IEEE Transactions on Systems Man and Cybernetics*, 7:326–340, 1977. 165, 168

[28] M. Ehrgott, J. Figueira, and S. Greco. *Trends in Multiple Criteria Decision Analysis*. Springer, 2010. 2, 8, 60

[29] J. Fieberg and K. Jenkins. Assessing uncertainty in ecological systems using global sensitivity analyses: a case example of simulated wolf reintroduction effects on elk. *Ecological Modelling*, 187:259–280, 2005. 52

[30] A. Forrester, A. Sobester, and A. Keane. *Engineering Design via Surrogate Modelling.* Wiley, 2008. 93, 114

[31] H. G. Gemuenden and J. Hauschildt. Number of alternatives and efficiency in different types of top-management decisions. *European Journal of Operational Research*, 22:178–190, 1985. 122

[32] M. S. Germain. Test for significance, 2007. 63

[33] V. Ginot, S. Gaba, R. Beaudouin, F. Aries, and H. Monod. Combined use of local and anova-based global sensitivity analyses for the investigation of a stochastic dynamic model: Application to the case study of an individual-based model of a fish population. *Ecological Modelling*, 193:479–491, 2006. 52, 154

[34] D. M. Hamby. A review of techniques for parameter sensitivity analysis of environmental models. *Environmental Monitoring and Asssessment*, 32:135–154, 1994. 153, 154

[35] W. Heinze. *Entwerfen von Verkehrsflugzeugen I.* Technical University of Braunschweig, 2005. 74

[36] J. Helton. Conceptual and computational basis for the quantification of margins and uncertainty. Technical report, SANDIA National Laboratories, 2009. 52, 60

[37] D. C. Howell. *Statistical Methods for Psychology.* Wadsworth, Cengage Learning, 2010. 61

[38] C. L. Hwang and A. S. Masud. *Multiple Objective Decision Making Methods and Applications.* Springer, 1979. 9

[39] C. L. Hwang and K. Yoon. *Multiple Attribute Decision Making Methods and Applications: A State of the Art Survey.* Springer, 1981. ix, xiii, 7, 8, 10, 11, 12, 18, 19, 26, 27, 29, 34, 38, 165, 167

[40] M. Janic and A. Reggiani. An application of the mutiple criteria decision making (mcdm) analysis to the selection of a new hub airport. *European Journal of Transport and Infrastructure Research*, 2:113–141, 2002. 3

[41] L. Jenkinson, P. Simpkin, and D. Rhodes. *Civil Jet Aircraft Design.* Butterworth Heinemann, 1999. 73, 74

[42] N. Kano, N. Seraku, F. Takahashi, and S. Tsuji. Attractive quality and must-be quality. *Journal of the Japanese Society for Quality Control*, April:39–48, 1984. 168

[43] R. L. Keeney and H. Raiffa. *Decision with Multiple Objectives: Preferences and Value Tradeoffs.* Cambridge University Press, 1993. 22

[44] M. R. Kirby. *A Methodology for Technology Identification, Evaluation, and Selection in Conceptual and Preliminary Aircraft Desgin.* PhD thesis, Aerospace Systems Design Laboratory, School of Aerospace Engineering, Georgia Institute of Technology, 2001. 3

[45] I. Kroo. *Aircraft Design: Synthesis and Analysis.* Stanford University, 2006. 84

[46] P. H. Kvam and B. Vidakovic. *Nonparametric Statistics with Applications to Science and Engineering.* Wiley, 2007. 62

[47] S. Lehner and W. Crossley. Combinational optimization to include greener technologies in a short-to-medium range commercial aircraft. In *The 26th Congress of International Council of Aeronautical Sciences (ICAS), Anchorage, Alaska*, 14-19 September 2008. 83

[48] Y. Li. *An Intelligent Knowledge-based Multiple Criteria Decision Making Advisor for Systems Design.* PhD thesis, Aerospace Systems Design Laboratory, School of Aerospace Engineering, Georgia Institute of Technology, 2007. 3, 33, 38

[49] Y. Li, D. Mavris, and D. DeLaurentis. The investigation of a decision making technique using the loss function. In *AIAA 4th Aviation Technology, Integration and Operation (ATIO) Forum*, 2004. 3

[50] A. Lovison and E. Rigoni. Adapative sampling with a lipschitz criterion for a accurate metamodeling. *Communications in Applied and Industrial Mathematics*, 1:110–126, 2010. 93

[51] R. Lowry. *Concepts and Applications of Inferential Statistics.* Vassar College, 1998. 62

[52] J. Lu, M. Quaddus, K.L.Poh, and R. Williams. The design of a knowledge-based guidance system for an intelligent multiple objective decision support system (imodss). In *10th Australasian Conference on Information Systems*, 1999. 35

[53] K. MacCrimmon. *An overview of Multi-Objective Decision Making.* The University of South Carolina Press, 1973. 34

[54] S. Marino, I. Hogue, C. Ray, and D. Kischner. A methodology for performing global uncertainty and sensitivity analysis in system biology. *Journal of Theoretical Biology*, 254:178–196, 2008. 60, 64

[55] M. D. Mckay, R. J. Beckman, and W. Conover. Comparison of 3 methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics*, 21:239–245, 1979. 64, 93

[56] K. Mehlhorn and P. Sanders. *Algorithms and data structures- the basic toolbox.* Springer, 2008. 53, 55

[57] MEI. Spearman rank correlation coefficient. `www.mei.org.uk/files/pdf/Spearmanrcc.pdf`, December 2007. 65

[58] F. Meller. Key buying factors and added value- a new approach to aircraft evaluation. In *DGLR Workshop-Aircraft Evaluation*, 1998. 1, 3

[59] P. Miettinen and R. Hmlinen. How to benefit from decision analysis in environmental life cycle assessment (lca). *European Journal of Operational Research*, 102:279–294, 1997. 165, 169

[60] A. S. Milani, A. Shanian, and C. Lahham. Using different electre methods in strategic planning in the presence of human behavioral resistance. *Journal of Applied Mathematics and Decision Sciences*, 2006:1–19, 2006. 7, 16, 17

[61] S. Mishra, N. Deeds, and G. Ruskauff. Global sensitivity analysis techniques for probabilistic ground water modeling. *Ground Water*, 47:727–744, 2009. 52, 67, 148

[62] D. C. Montgomery and G. C. Runger. *Applied Statistics and Probability for Engineers.* John Wiley and Sons, Inc., 2006. 46, 63, 137

[63] R. H. Myers and D. C. Montgomery. *Response Surface Methodology: Process and Product Optimization Using Designed Experiments.* Wiley, 2005. 93, 96

[64] K. L. Poh. A knowledge-based guidance system for multi-attribute decision making. *Artificial Intelligence in Engineering*, 12:315–326, 1998. 35

[65] D. Raymer. *Enhancing Aircraft Conceptual Design using Multidisciplinary Optimization.* PhD thesis, Royal Institute of Technology, 2002. 74

[66] F. Roman, N. Rolander, M. G. Fernndez, B. Bras, J. Allen, F. Mistree, P. Chastang, P. Dpinc, and F. Bennis. Selection without reflection is a risky business... In *10th AIAA/SSMO Multidisciplinary Analysis and Optimization Conference, 30 August-1 September 2004, Albany, New York*, 2004. 34, 35

[67] B. Roy. Decision-aid and decision-making. *European Journal of Operational Research*, 45:324–331, 1990. 8

[68] B. Roy. The outranking approach and the foundations o f electre methods. *Theory and Decision*, 31:49–73, 1991. xiii, 7, 12, 16, 17

[69] A. Rubin. *Statistics for Evidence-Based Practice and Evaluation.* Thomson, 2007. 63

[70] T. L. Saaty. *The Analytic Hierarchy Process.* University of Pittsburg, 1988. xiii, xv, 19, 20, 165, 166, 167

[71] J. Sacks, W. Welch, T. Mitchell, and H. Wynn. Design and analysis of computer experiments. *Statistical Science*, 4:409–435, 1989. 114

[72] A. Saltelli, S. Tarantola, and K. Chan. A quantitative model-independent method for global sensitivity analysis of model output. *Technometrics*, 41:39–56, 1999. 52, 60

[73] P. Sen and J. B. Yang. *Multiple Criteria Decision Support in Engineering Design.* Springer, 1998. ix, 8, 34

[74] D. Sheskin. *Handbook of Parametric and Nonparametric Statistical Procedures.* Chapman and Hall/CRC, 2004. 61, 63

[75] T. Simpson, T. Mauery, J. Korte, and F. Mistree. Comparison of response surface and kriging models for multidiciplinary design optimization. In *AIAA*, 1998. 114

[76] A. Strohmayer and D. Schmitt. Scenario based aircraft design evaluation. In *International Council of the Aeronautical Sciences (ICAS)*, 2000. 2

[77] X. Sun and Y. Li. An intelligent multi-criteria decision support system for systems design. In *13th Multidisciplinary Analysis and Optimization (MAO) and 10th Aviation Technology Integration and Operations (ATIO) Conference, Texas, USA*, 13-15 September 2010. 33, 38

[78] G. Taguchi, S. Chowdhury, and Y. Wu. *Taguchi's Quality Engineering Handbook*. Wiley, 2005. 51

[79] A. Tecle. Selecting a multicriterion decision making technique for watershed resources management. *Water Resources Bulletin*, 28:129–140, 1992. 34

[80] M. Thurber. Aviation International News 2010 Product Support Survey, August 2010. x, xiv, 119, 120

[81] E. Triantaphyllou. *Multi-Criteria Decision Making Methods: A comparative Study*. Kluwer Academic Publishers, 2000. 34, 53

[82] T. Wang and P. Ji. The understanding customer needs through quantitative analysis of kanos model. *International Journal of Quality and Reliability Management*, 27:173–184, 2010. 3

[83] N. Wirth. *Algorithms and data structures*. Federal Institute of Technology, 2004. 53

[84] S. Zanakis, A. Solomon, N. Wishart, and S. Dublish. Multi-attribute decision making: A simulation comparison of select methods. *European Journal of Operational Research*, 107:507–529, 1998. 122

[85] M. Zeleny. *Multiple Criteria Decision Making*. McGraw-Hill Book Company, 1982. 8

[86] Y. Zhang and A. Rundell. Comparative study of parameter sensitivity analyses of the tcr-activated erk-mapk signalling pathway. *IEE Proceedings of System Biology*, 153:201–211, 2006. 52, 154

[87] C. Zopounidis and P. Pardalos. *Handbook of Multi-Criteria Analysis*. Springer, 2005. 1, 2, 8, 34

[88] R. Zultner and G. Mazur. The kano model: Recent developments. In *Transactions from The Eighteenth Symposium on Quality Function Deployment*, 2006. 165

Appendix A

Preference Information Elicitation Techniques

Most MCDA methods require preference information about the relative importance of each criterion. It is usually given by a set of weighting factors. This Appendix introduces several typical weight elicitation techniques: direct assignment method (39), eigenvector method (70), entropy method (39), Simple Multi-Attribute Rating Technique (SMART)(27), Kano's model (88), and distance-to-target method (59).

A.1 Direct Assignment Method

In this method, the DM directly assigns numbers to represent the relative importance of one criterion over others. For instance, a ten-point scale can be chosen with calibration that 0 stands for extremely unimportant criterion, while 10 stands for extremely important one, as shown in Table A.1.

This method is popular because of its simplicity. However, it should be noted that the numerical assignment is arbitrary, and this type of scaling assumes that a scale value of 9.0 is three times as favorable as a scale value of 3.0. Besides, it also assumes that the difference between *low* and *high* is the same as the difference between *average* and *very high*. In complex decision making problems, it is rather difficult to precisely assign directly weights for all criteria even for an experienced DM.

Table A.1: Direct Assignment Method with a Ten-point Scale

Criterion evaluation	Value
Extremely low	0
Very low	1.0
Low	3.0
Average	5.0
Very high	9.0
Extremely high	10.0

A.2 Eigenvector Method

The eigenvector method is an analytical way of eliciting preference information of criteria in Analytical Hierarchy Process (AHP) (70). This method uses pairwise comparisons between criteria represented by a comparison matrix M, the relative weights of criteria can be obtained by solving the eigenvalue function, as shown in Equation A.1 (70).

$$M * W = \lambda_{max} * W \tag{A.1}$$

where λ_{max} is the maximum eigenvalue of the comparison matrix M, the weights of criteria are the normalized eigenvector $W = [w_1, w_2, ..., w_n]^T$ corresponding to the maximum eigenvalue.

In most decision making problems, the eigenvalue function is solved to evaluate the priorities of different criteria. In AHP, the consistency of the weights is assessed by Consistency Ratio (CR), as shown in Equation A.2.

$$CR = \frac{CI}{RI} \tag{A.2}$$

where Consistency Index (CI) is calculated by Equation A.3.

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{A.3}$$

Random Consistency Index (RI) is an average value derived from a large sample of reciprocal matrices having all elements varying from 1/9 to 9. Table A.2 lists the RI for up to ten elements (70).

Table A.2: Random Consistency Index (RI)(70)

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.89	1.11	1.25	1.35	1.40	1.45	1.49

In general, CR of 0.1 or less is considered acceptable. In order to maintain reasonable consistency when deriving weights from pairwise comparisons, it is suggested that the number of elements being considered should be less than nine.

A.3 Entropy Method

The entropy method provides an alternative way of assigning weights when the input evaluation data of a decision making problem is represented by decision matrix, the weights of criteria w_j can be calculated by Equation A.4 (39).

$$\begin{aligned}
w_j &= \frac{d_j}{\sum\limits_{j=1}^{n} d_j}, \;\; \forall j \\
d_j &= 1 - E_j, \;\; \forall j \\
E_j &= -\frac{1}{\ln m} \sum_{i=1}^{n} p_{ij} \ln p_{ij}, \;\; \forall j \\
p_{ij} &= \frac{x_{ij}}{\sum\limits_{i=1}^{n} x_{ij}}, \;\; \forall i, j
\end{aligned} \tag{A.4}$$

where p_{ij} is the value of the j-th criterion $(i = 1, 2, ..., m, j = 1, 2, ..., n)$, E_j is the entropy of the j-th criterion value, d_j is the degree of diversity of the information involved in the j-th criterion value.

The entropy method helps to investigate contrasts between sets of data, that is, the weight of a criterion is small when all the alternatives have similar values on the criterion. In other words, a criterion does not contribute much when the criterion has similar values for all alternatives.

A.4 SMART

Simple Multi-Attribute Rating Technique (SMART) was originally developed as a whole process of rating alternatives and weighting criteria (27). The weights are obtained in two steps:

- Firstly, the DM ranks the importance of the changes in the criteria from the worst criterion levels to the best criterion levels;
- Then, make ratio estimates of the relative importance of each criterion relative to the one ranked lowest in importance.

The second step usually begins with assigning ten points to the least important criterion. The relative importances of the other criteria are then evaluated by giving them points from ten upwards.

A.5 Kano's Model

Kano's model provides a way of classifying importance among the attributes of alternatives (42), where three types of product attributes were distinguished: must-be attributes, one-dimensional attributes, and attractive attributes.

- **Must-be attributes**: The must-be attributes are the basic requirement of the product. The consumer regards these attributes as prerequisites. Their fulfillment will not increase consumer's satisfaction; however, if the product does not have these attributes, the customer will become extremely dissatisfied.
- **One-dimensional attributes**: The one-dimensional attributes have proportional satisfaction degree with regard to their fulfillment level. The consumer has more satisfaction with better attributes.
- **Attractive attributes**: The attractive attributes are unique selling points of the product. The consumer will not feel dissatisfaction without them, however, their fulfillment greatly enhance the consumer's expectation and satisfaction.

Each attribute type described above influences customer satisfaction in a different way, as shown in Figure A.1. As time passes by, the attractive attributes will evolve into one-dimensional ones, and the one-dimensional attributes will evolve into must-be ones, and new attractive attributes will emerge.

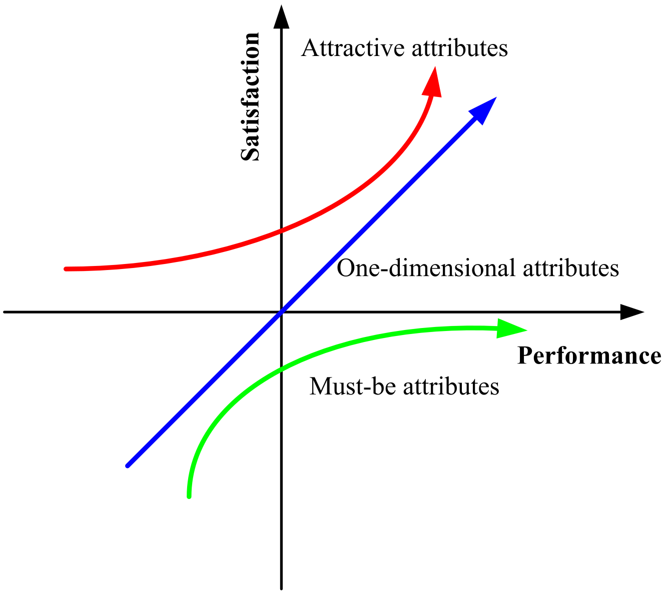


Figure A.1: Attributes Classification in Kano's Model (9)

A.6 Distance-to-target Method

The distance-to-target method is widely applied in the field of Life Cycle Assessment (LCA), which describes the environmental impacts associated with a product, process, or service by multi-attribute product evaluations (59). The distance-to-target method derives the weights from the distance between the current levels of the criteria and the future target values (3).

Appendix B

User Guide of an Intelligent Multi-Criteria Decision Support System

An intelligent knowledge-based decision support system is developed in MATLAB 2010 (*www.mathworks.com*). The main interface is illustrated in Figure B.1. The intelligent knowledge-based decision support system has the capabilities to select the most appropriate method, use specific method to solve given problem, and perform uncertainty assessment in the decision analysis process. The user guide for each desired task is described in detail as follows.

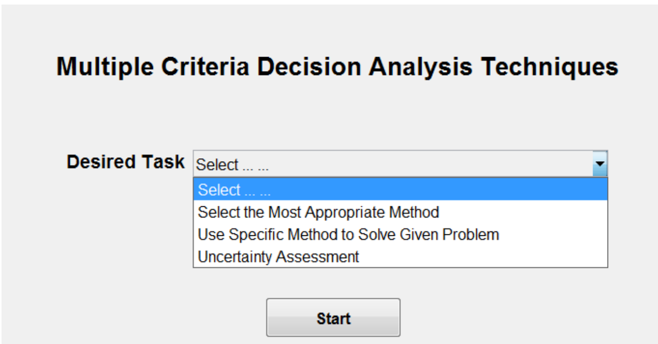


Figure B.1: Main Interface of an Intelligent Multi-Criteria Decision Support System

B.1 Select the Most Appropriate Method

When the DM wants to select the most appropriate method, the DM related requirements and problem related requirements needs to be defined, respectively. The interface of DM related characteristics is illustrated in Figure B.2 and is summarized in Figure B.3. If the summary is not satisfying, then the DM can simply click *Back to User Definition* and redefine the requirements; otherwise, the DM can click *Confirm and Proceed* to move on to the next step.

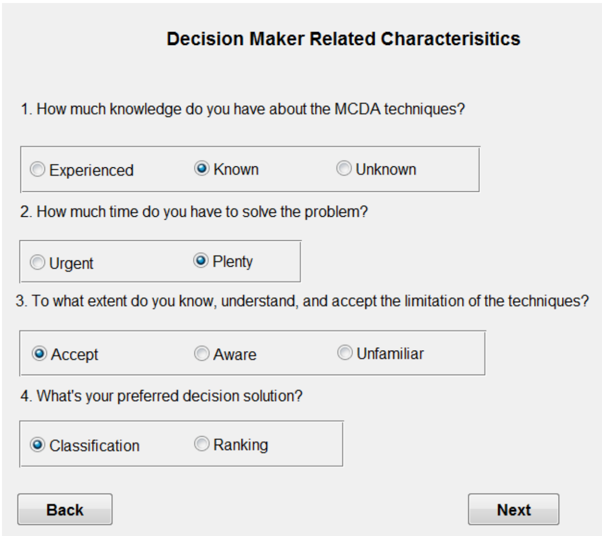


Figure B.2: Interface of Decision Maker Related Characteristics

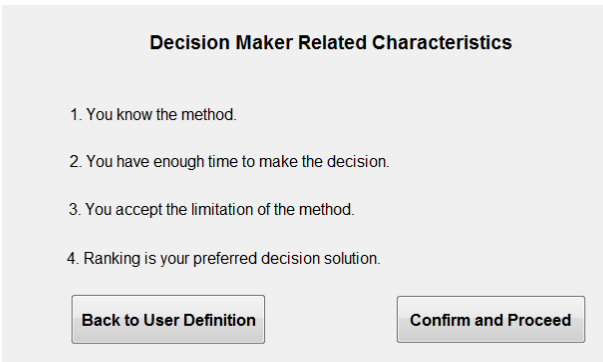


Figure B.3: Summary of Decision Maker Related Characteristics

When the DM is experienced about the MCDA techniques, the interface of all sixteen widely used MCDA methods is presented. This interface will be discussed in Section B.2. Thus, the DM can choose one preferred method to solve given problem.

The interface of problem related characteristics is most important, where the appropriateness score for each MCDA method is obtained. It is illustrated in Figure B.4 and is summarized in Figure B.5. If the summary is not satisfying, the DM can simply click *Back to User Definition* and redefine the requirements; otherwise, the DM can click *Confirm and Proceed* and the ranking of the MCDA methods with appropriateness scores is shown in Figure B.6.

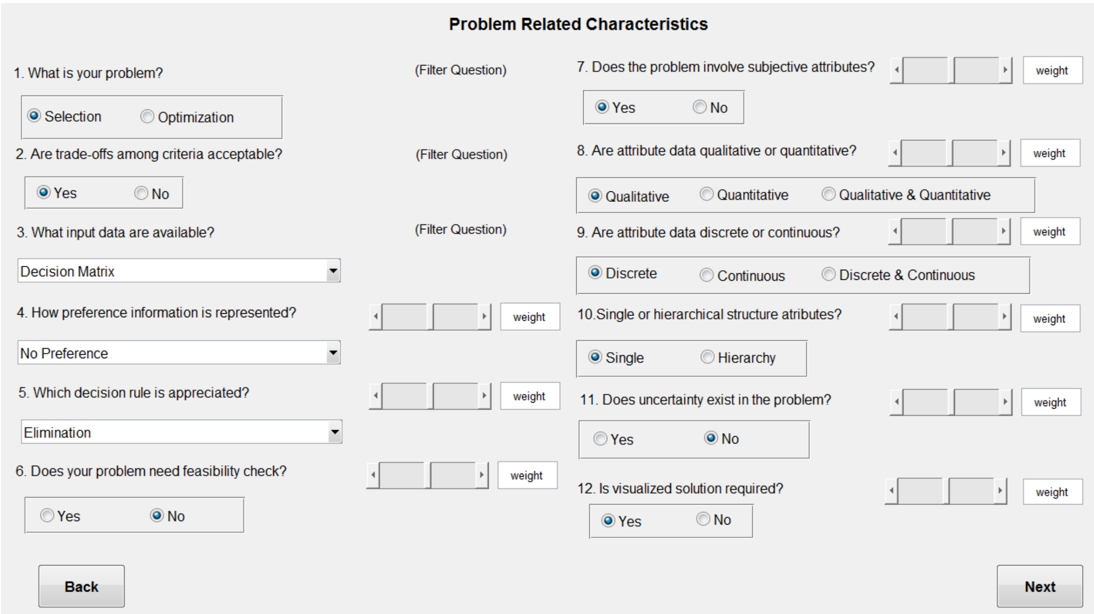


Figure B.4: Interface of Problem Related Characteristics

The DM can simply click the name of the most appropriate method, and methodology instructions will be shown to guide the DM to get the final solution. In addition, the mathematical calculation steps are also built in the decision support system. Thus, for evaluation decision making problems, the DM can input the data according to the instruction, and get the final results by clicking one corresponding button. For instance, methodology instructions of the Dominance method are illustrated in Figure B.7.

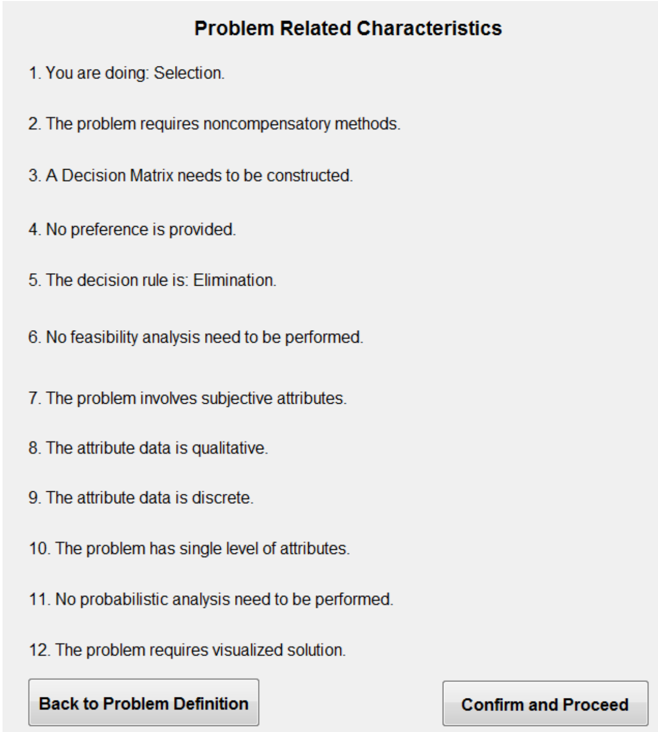


Figure B.5: Summary of Problem Related Characteristics

Attention should be paid that inconsistent input for the three filter questions will be rectified by the intelligent multi-criteria decision support system automatically. For instance, since compensation is always allowed in the optimization process, thus, if the DM selects the MCDA methods for optimization, all non-compensatory MCDA methods which cannot offer scores will be excluded. Even if the DM selects optimization for the first filter question and non-compensatory for the second filter question, the system will rectify the conflicting input by offering compensatory MCDA methods for solving optimization problem.

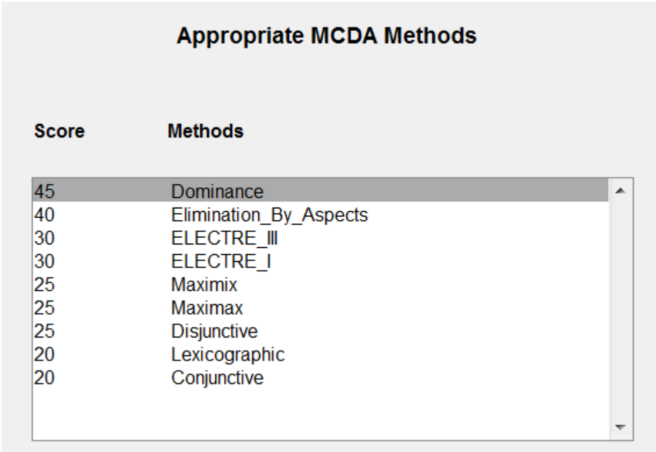


Figure B.6: Ranking of MCDA Methods with Appropriateness Scores

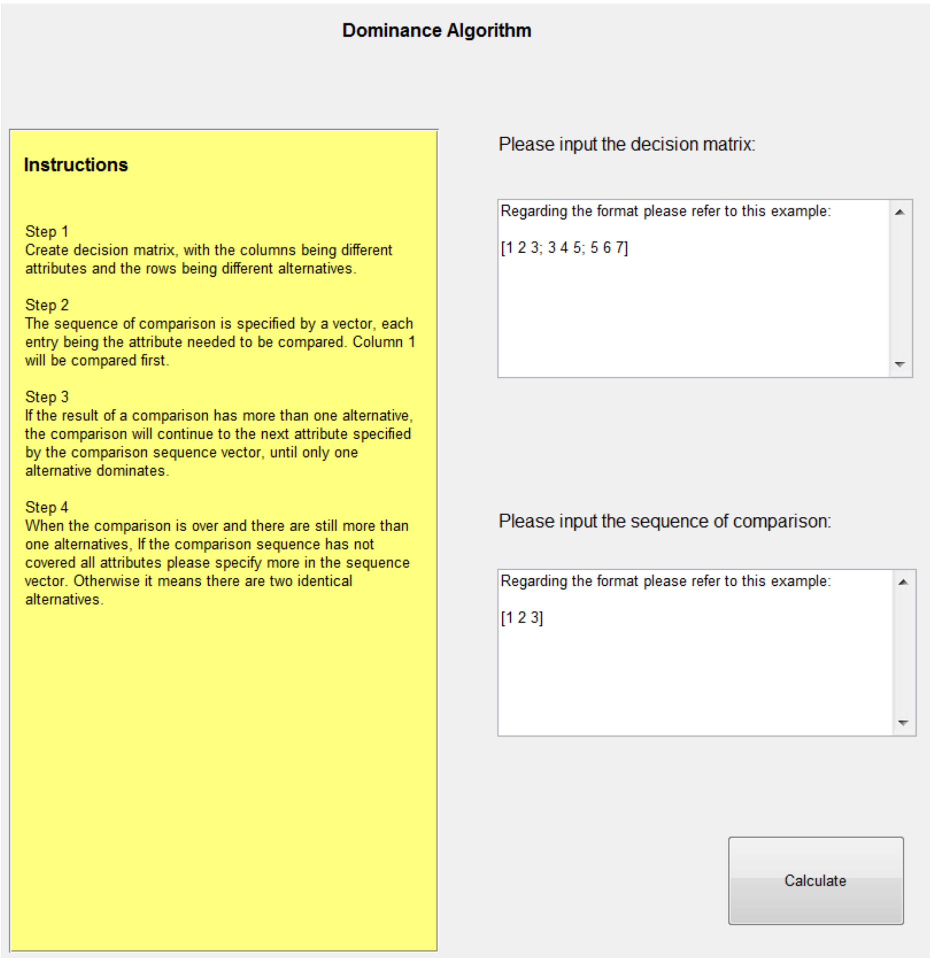


Figure B.7: Methodology Instructions for the Dominance Method

B.2 Use Specific Method to Solve Given Problem

When the DM wants to use specific method to solve a given problem, the interface of all sixteen widely used MCDA methods are listed in Figure B.8. As discussed previously, the DM can simply click the name of the most appropriate MCDA method, and methodology instructions will be shown to provide guidance to the DM how to get the final solution by using the selected method.

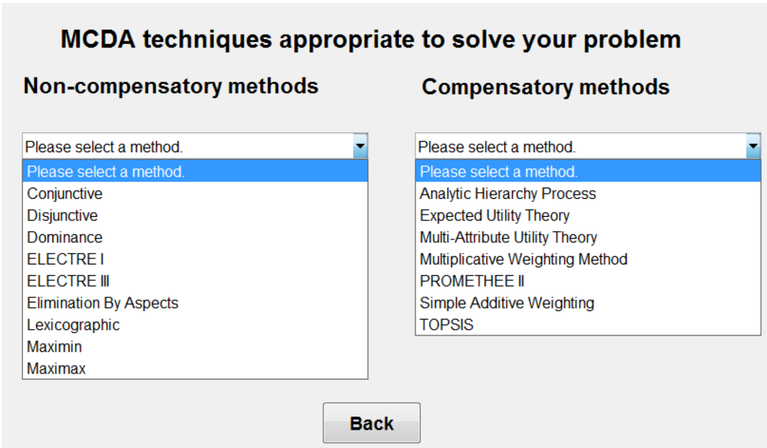


Figure B.8: Sixteen MCDA Methods List

B.3 Uncertainty Assessment

When the DM wants to perform uncertainty assessment, the interface of the uncertainty assessment module is illustrated in Figure B.9. In the uncertainty assessment module, the DM can simply go through the uncertainty assessment process according to the instructions. In addition, the detailed mathematical calculation steps for four MCDA techniques: SAW, multiplicative weighting method, TOPSIS, and ELECTRE I, are also built in the uncertainty assessment module, which highly facilitates the uncertainty assessment in the decision analysis process.

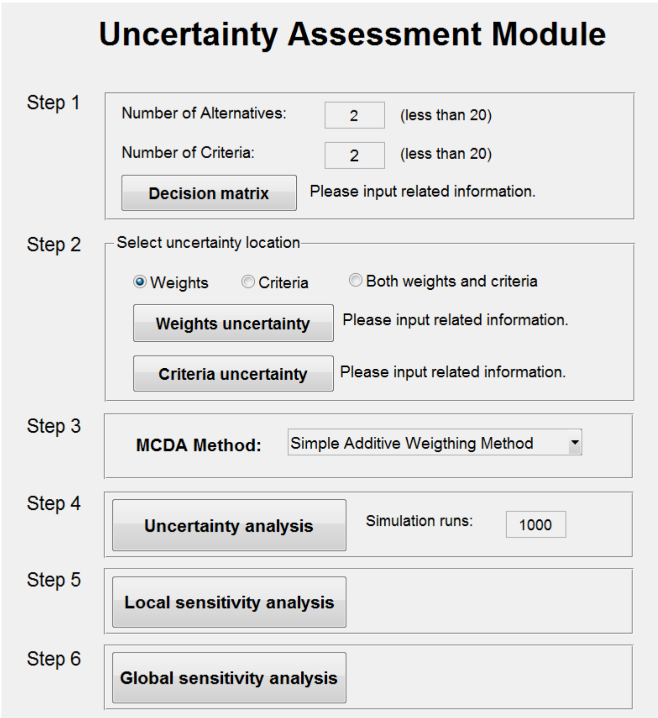


Figure B.9: Interface of Uncertainty Assessment Module

Appendix C

Additional Figures

C.1 Parametric Studies of Design Criteria

Parametric studies for aspect ratio, reference area, cruise Mach number, and fuselage diameter in the aircraft conceptual design tool (VAMPzero) are presented in Figure C.1, Figure C.2, Figure C.3 and Figure C.4, respectively.

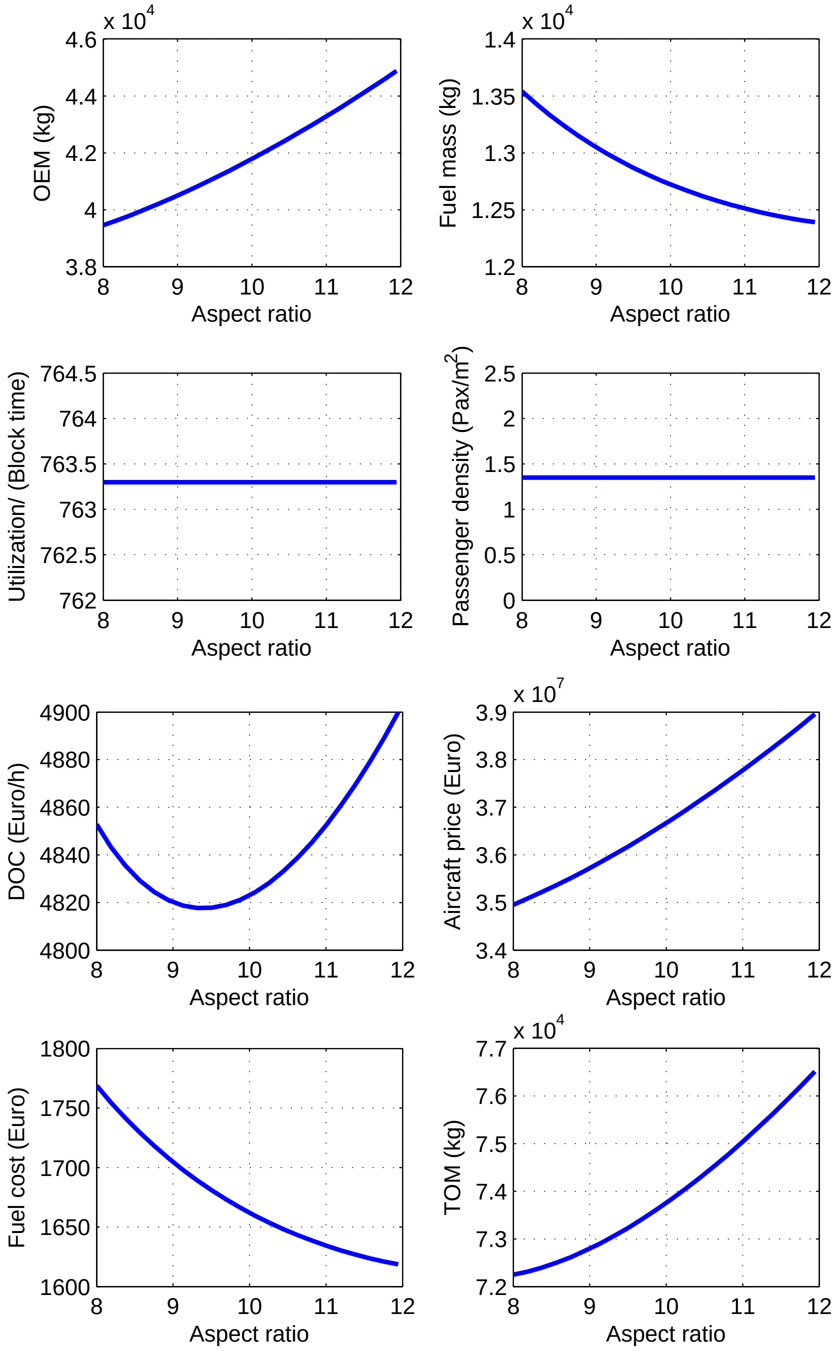


Figure C.1: Parametric Study of Aspect Ratio versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

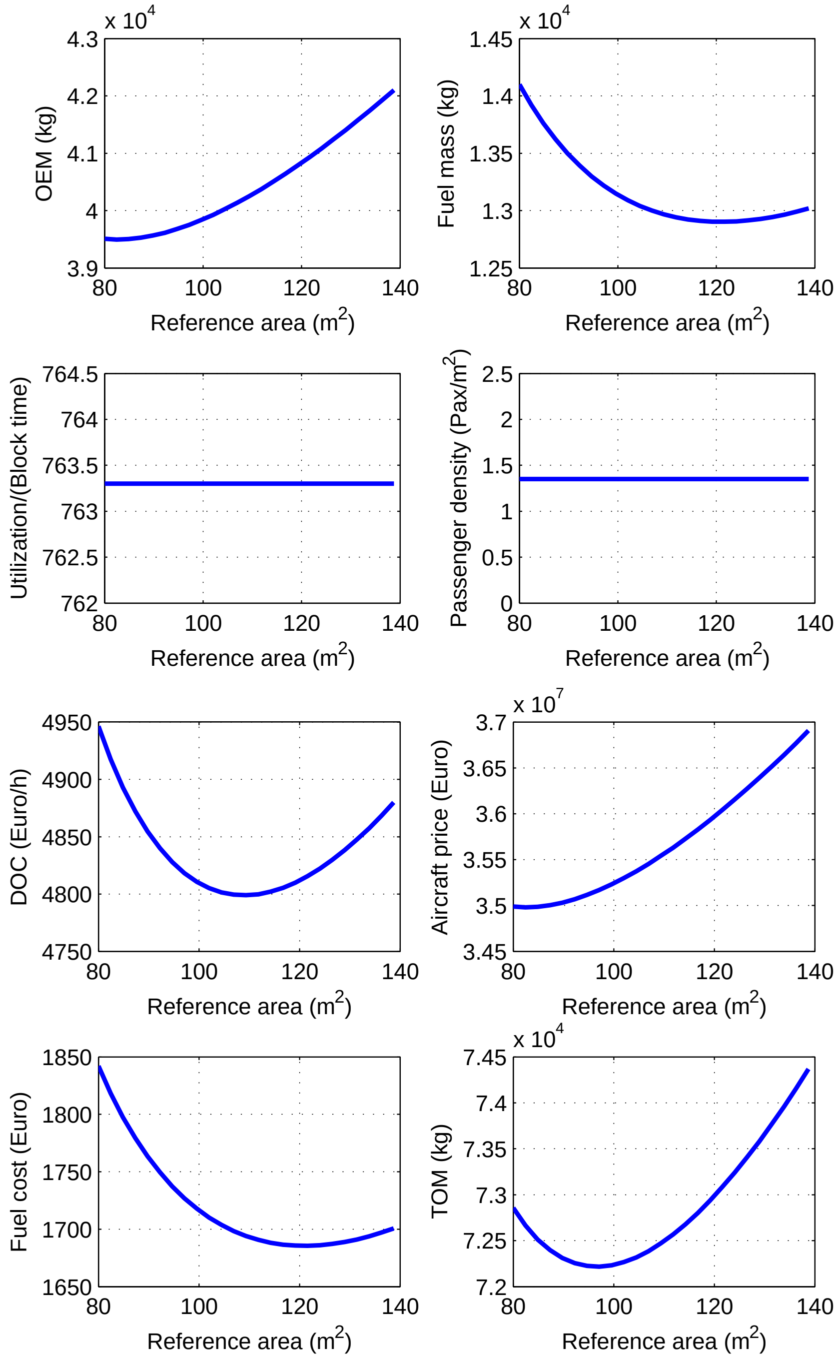


Figure C.2: Parametric Study of Reference Area versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

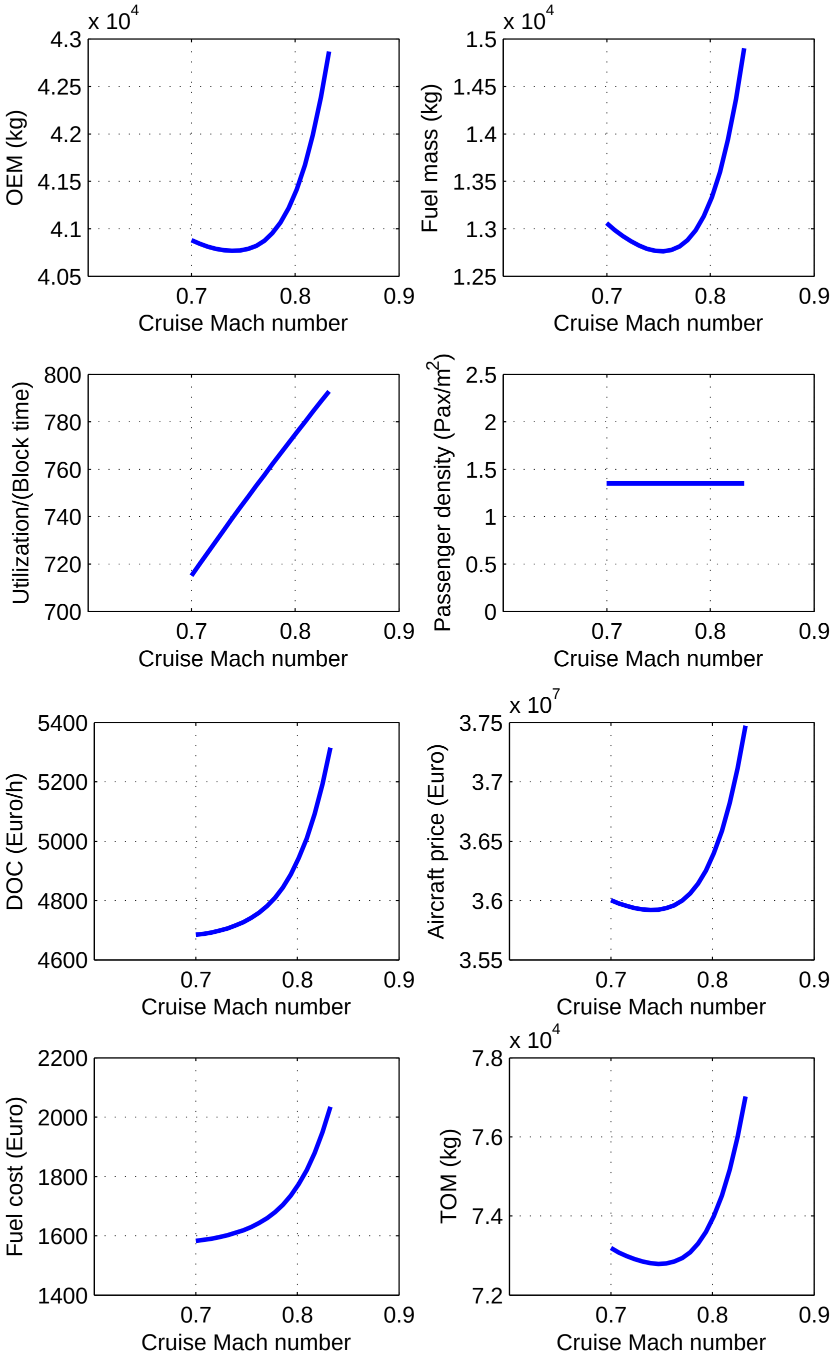


Figure C.3: Parametric Study of Cruise Mach Number versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

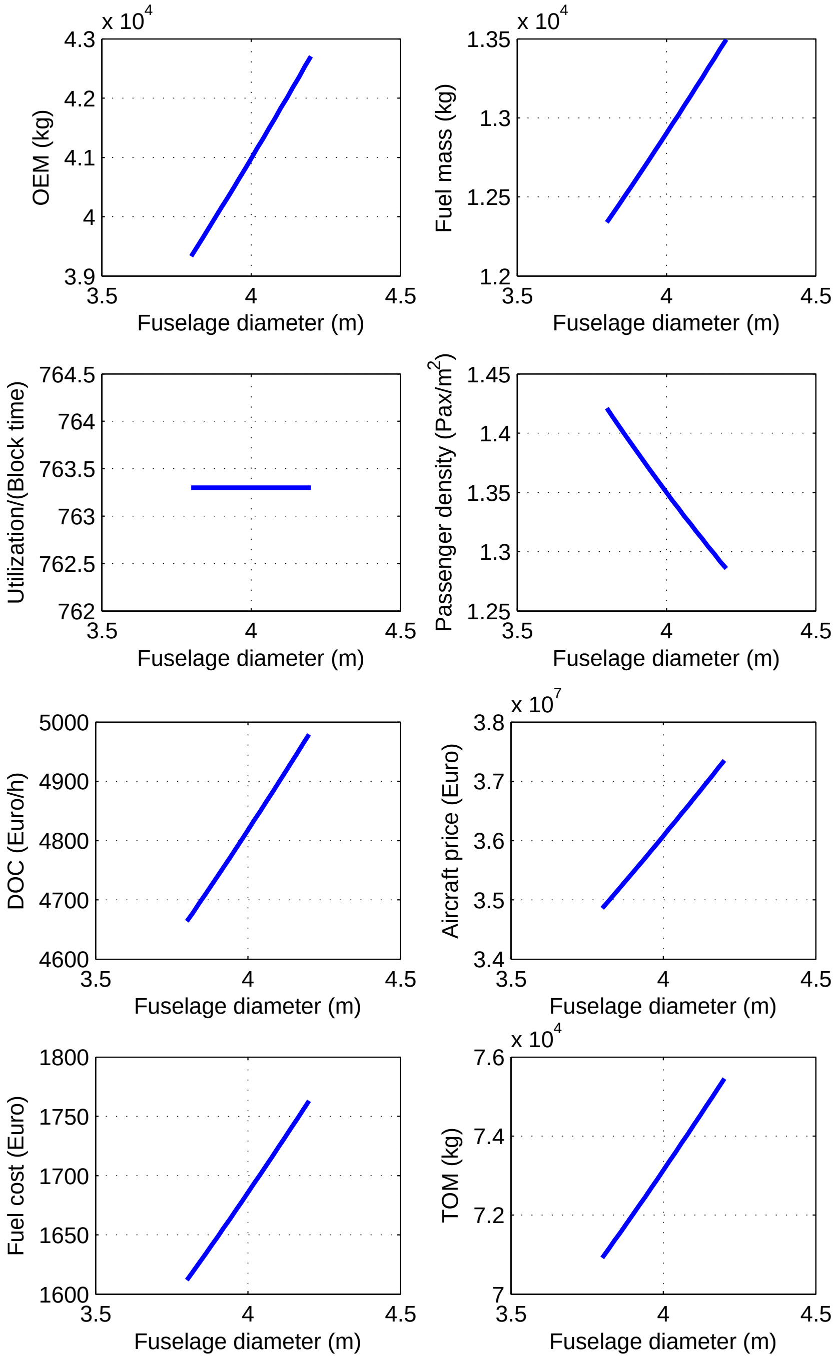


Figure C.4: Parametric Study of Fuselage Diameter versus OEM, Fuel Mass, Utilization/(Block time), Passenger Density, DOC, Aircraft Price, Fuel Cost, and TOM

C.2 Distributions of Design Criteria with Uncertainty Variation

10,000 Monte Carlo simulations are conducted through the developed surrogate models for four design criteria with parameter μ_W and $\sigma_{W_i}(i = 1, 2, ..., 9)$, as presented in Table 5.15 in Section 5.5. The distributions for fuel mass, utilization/(block time), and passenger density are shown in Figure C.5, Figure C.6, and Figure C.7, respectively.

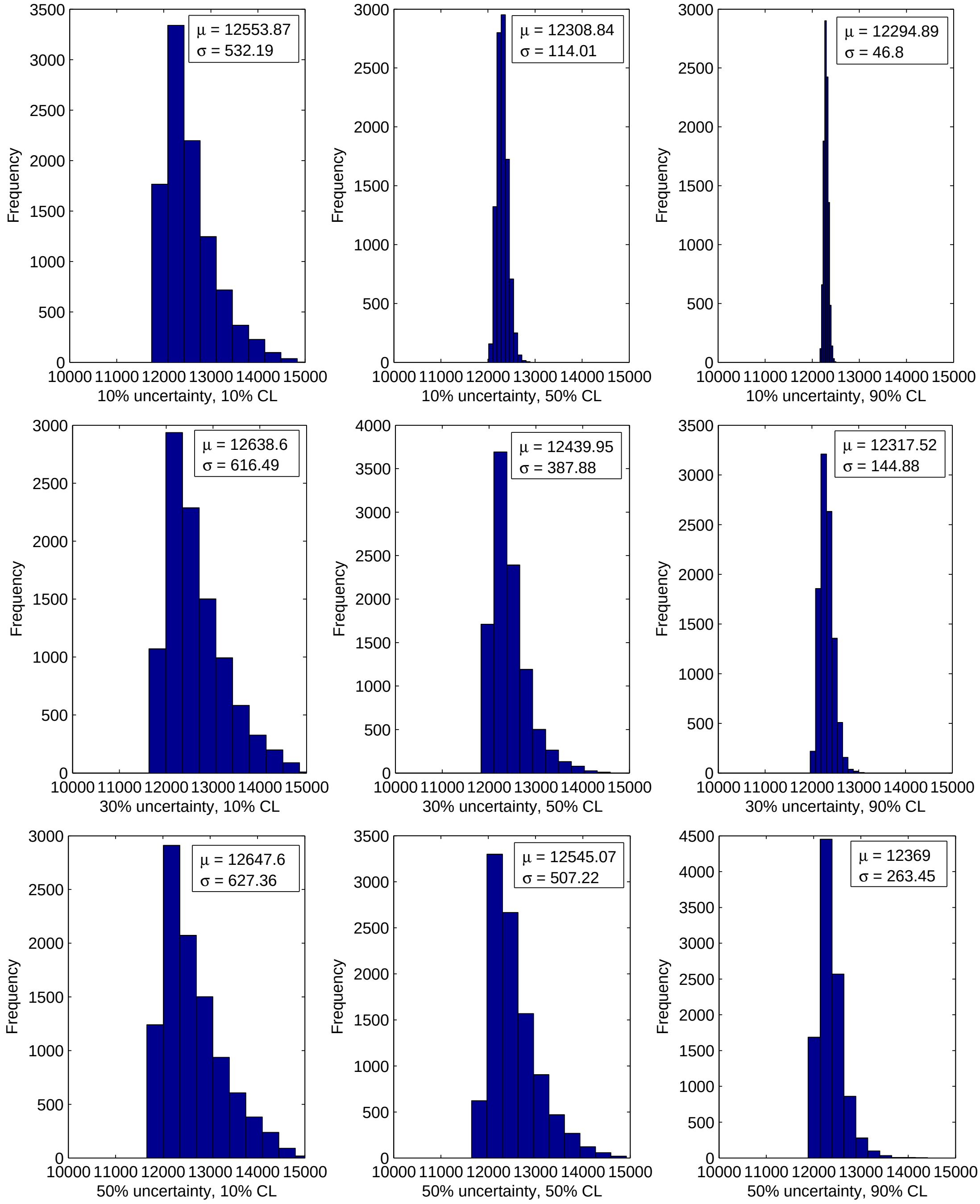


Figure C.5: Uncertainty Variation for Fuel Mass

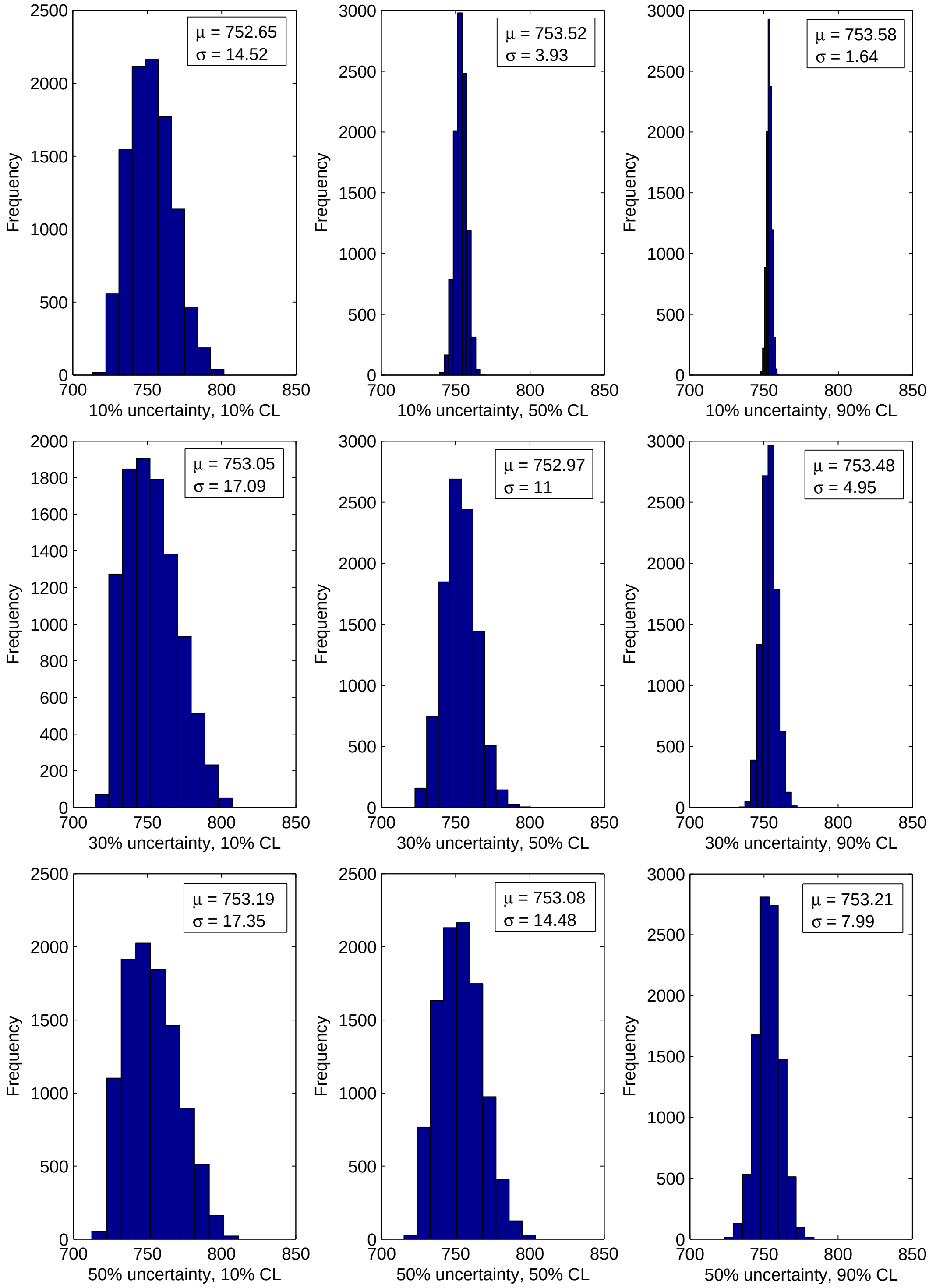


Figure C.6: Uncertainty Variation for Utilization/(Block time)

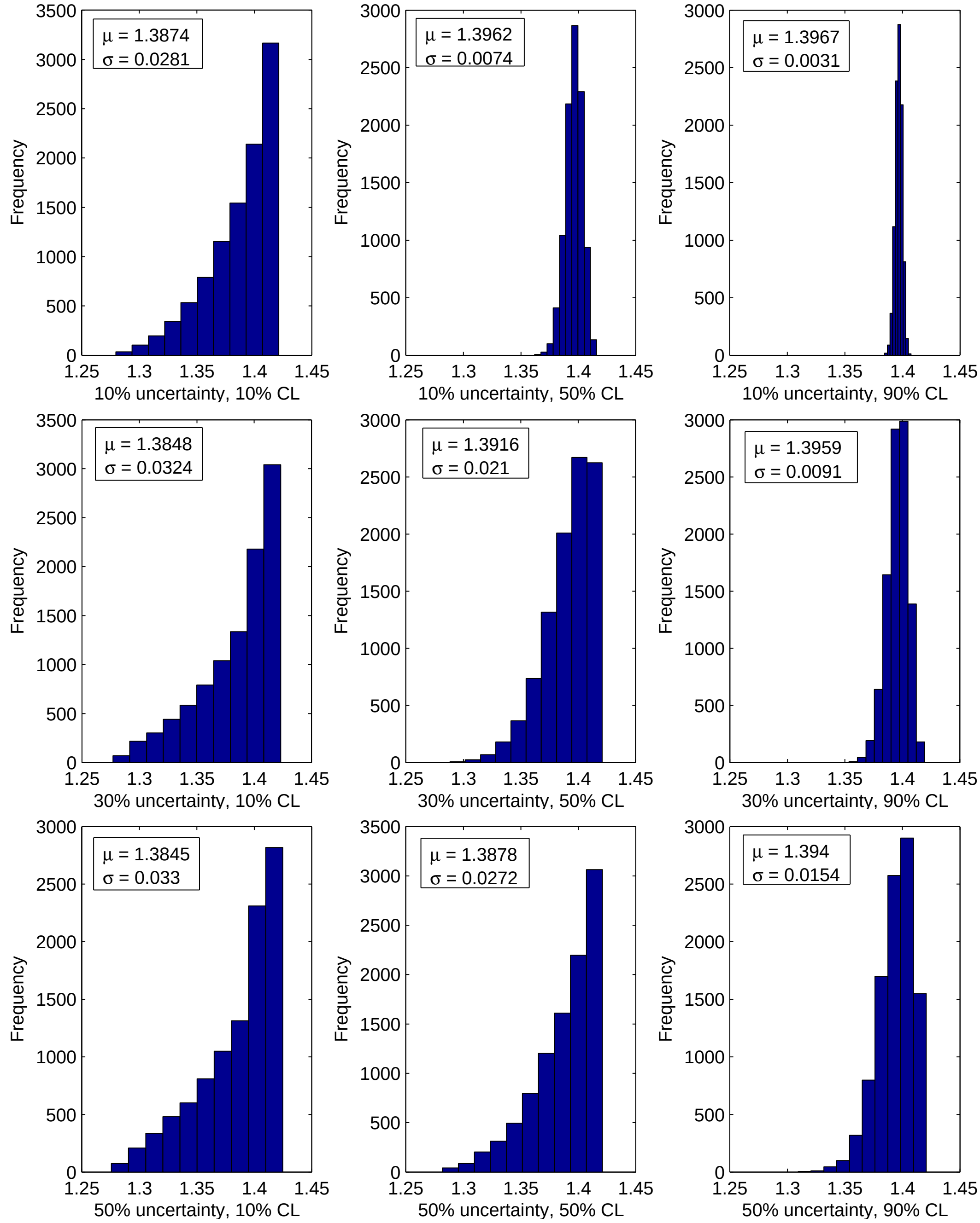


Figure C.7: Uncertainty Variation for Passenger Density

C.3 Interactive Weighting Plots for Business Aircraft Evaluation

The main idea of the interactive sensitivity analysis for weighting factors is to vary the weight of one criterion from 0 to 100%, while keeping the weighting factors of other criteria the same proportion as in the original setting. In the business aircraft evaluation problem using ELECTRE I, the interactive weighting plots for C_2 to C_7 are presented in Figure C.8, Figure C.9, Figure C.10, Figure C.11, Figure C.12, and Figure C.13, respectively, where *Non.* represents non-dominated alternative, and *Dom.* represents dominated alternative.

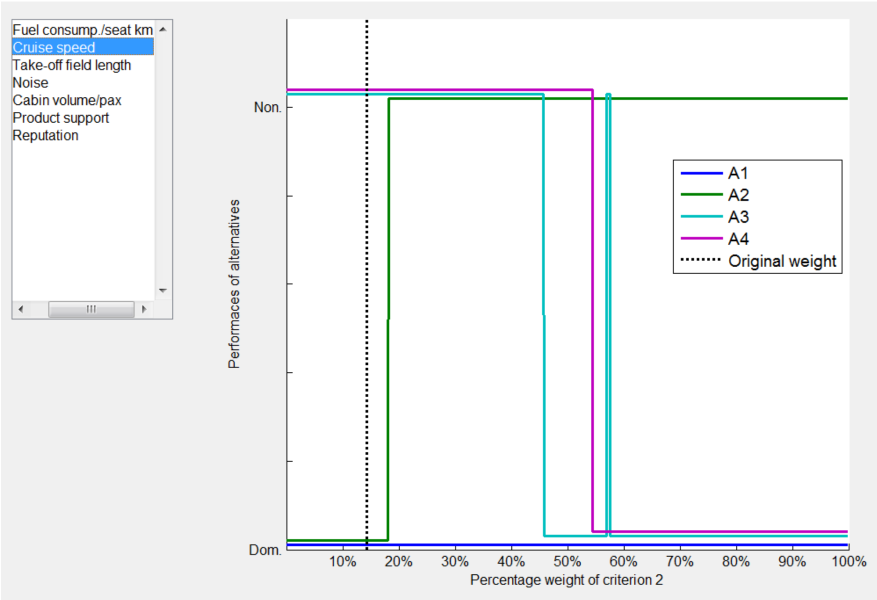


Figure C.8: Interactive Weighting Plot for Criterion 2

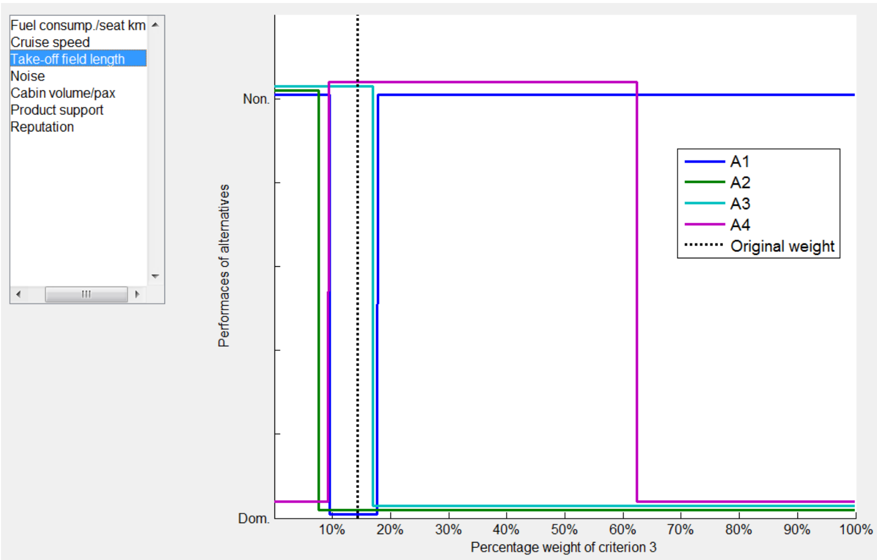


Figure C.9: Interactive Weighting Plot for Criterion 3

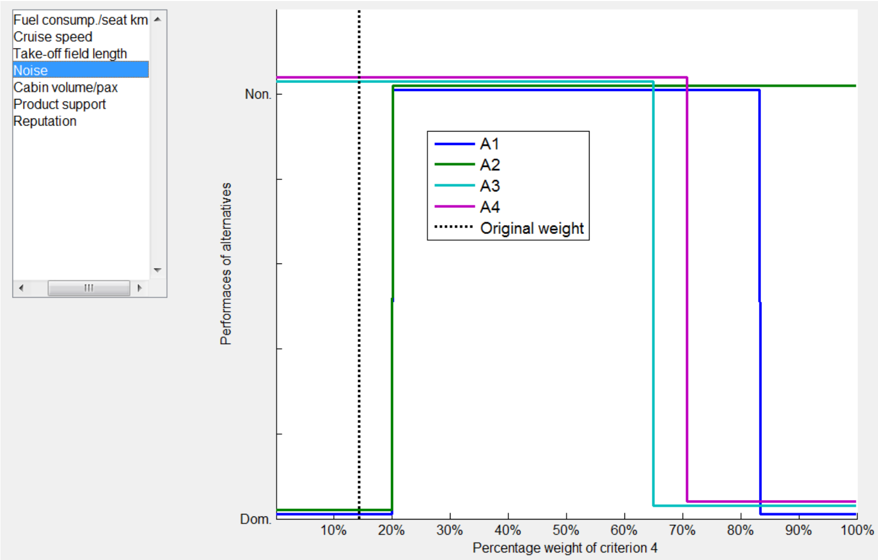


Figure C.10: Interactive Weighting Plot for Criterion 4

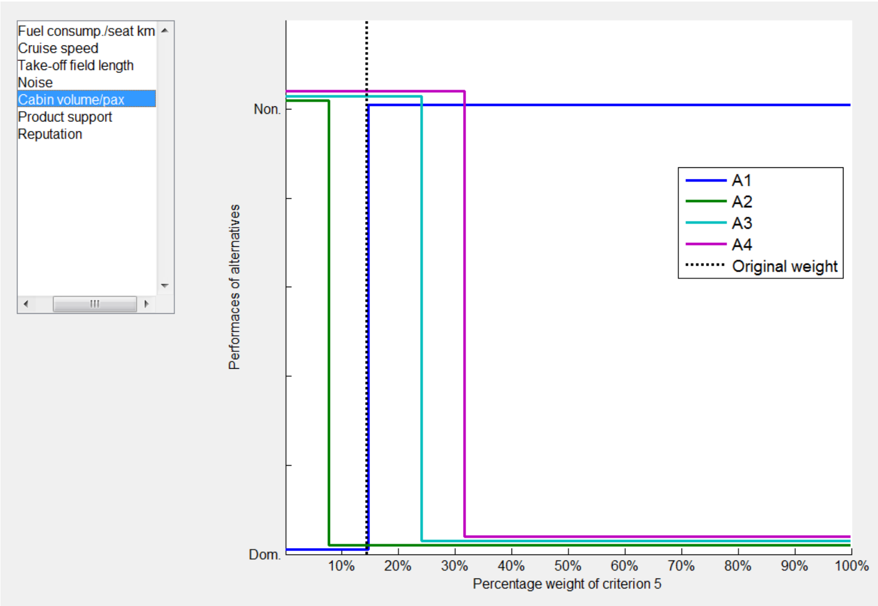


Figure C.11: Interactive Weighting Plot for Criterion 5

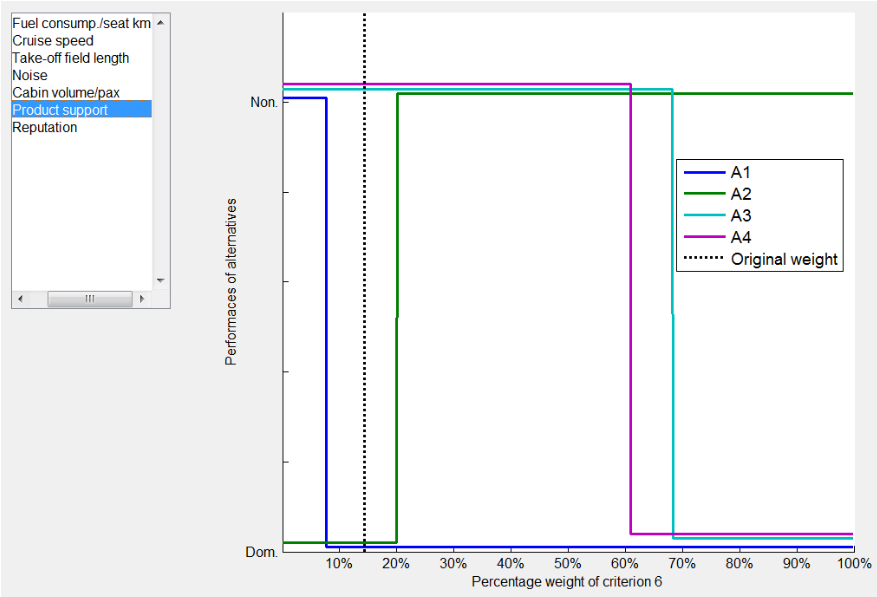


Figure C.12: Interactive Weighting Plot for Criterion 6

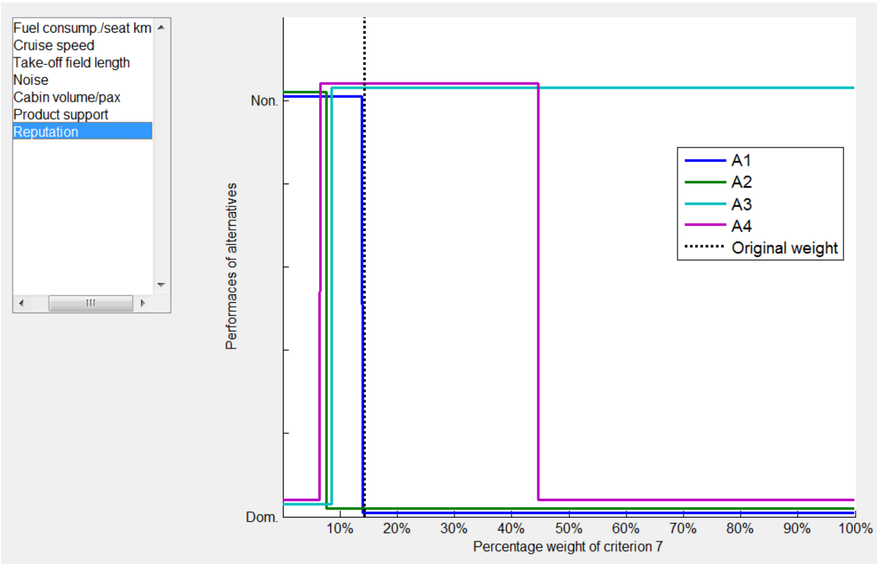


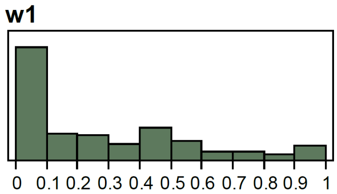
Figure C.13: Interactive Weighting Plot for Criterion 7

Appendix D

Data Sources

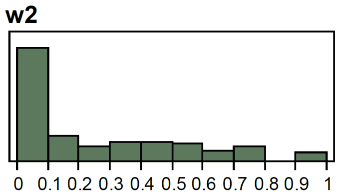
D.1 Data for Surrogate Model Construction in terms of Weighting Factors

One hundred sets of weighting factors are generated by the modified LHS with Dirichlet distribution. Histograms of the one hundred sets of weighting factors are depicted in Figure D.1. The corresponding values of the four design criteria (OEM, fuel mass, utilization/(block time), and passenger density) are listed in Table D.1.



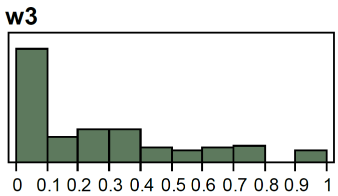
Moments

Mean	0.263379
Std Dev	0.2834573
Std Err Mean	0.0283457
Upper 95% Mean	0.3196231
Lower 95% Mean	0.2071349
N	100



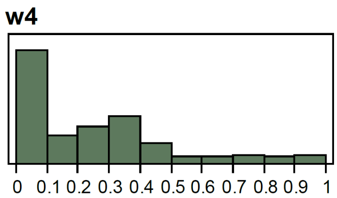
Moments

Mean	0.247492
Std Dev	0.2739894
Std Err Mean	0.0273989
Upper 95% Mean	0.3018574
Lower 95% Mean	0.1931266
N	100



Moments

Mean	0.252911
Std Dev	0.275488
Std Err Mean	0.0275488
Upper 95% Mean	0.3075738
Lower 95% Mean	0.1982482
N	100



Moments

Mean	0.236206
Std Dev	0.249093
Std Err Mean	0.0249093
Upper 95% Mean	0.2856314
Lower 95% Mean	0.1867806
N	100

Figure D.1: Histograms of One Hundred Sets of Weighting Factors Generated by Modified LHS with Dirichlet Distribution

Table D.1: One Hundred Sets of Weighting Factors Generated by Modified LHS with Dirichlet Distribution and Design Criteria Values

Set	w_1	w_2	w_3	w_4	OEM	Fuel Mass	Utilization/ (Block time)	Passenger Density
1	0.4333	0.0176	0.3719	0.1772	37895.82	13291.65	766.7125	1.4062
2	0.0269	0.9322	0.0407	0.0001	41428.86	11883.51	740.8892	1.4062
3	0.1942	0.798	0.0077	0	40342.85	11976.27	727.2155	1.4062
4	0.0231	0.0886	0.7454	0.1429	45184.84	13279.81	794.9339	1.3564
5	0.0017	0.008	0.088	0.9023	47650.82	13842.98	794.4662	1.2981
6	0	0.0498	0.0302	0.9199	46678.28	12649.03	740.8611	1.2981
7	0.0033	0.5557	0.0032	0.4379	44164.19	12152.47	732.6296	1.3553
8	0.7703	0.0002	0.2081	0.0215	37406.54	13364.7	751.3264	1.4062
9	0.0012	0.998	0.0002	0.0006	43910.36	11868.22	734.5968	1.4062
10	0.1196	0.0007	0.0129	0.8668	39874.9	14846.64	715.0679	1.2981
11	0.292	0.3525	0.3555	0.0001	39293.32	12329.55	755.8107	1.4062
12	0.268	0.4633	0.2516	0.0171	39574.7	12161.37	744.7317	1.4062
13	0.9818	0.001	0.0007	0.0165	37279.76	13292.7	731.715	1.4062
14	0.3792	0	0.6033	0.0174	38533.39	13335.2	780.4182	1.4062
15	0.0059	0.1591	0.216	0.619	47934.89	12751.37	756.5135	1.2981
16	0.0135	0.7722	0.2143	0	44079.11	11889.21	760.8023	1.4062
17	0.7002	0.2848	0.0142	0.0007	37908.16	12719.78	728.1471	1.4062
18	0	0.3933	0.3362	0.2705	45670.25	12112.59	762.0093	1.3776
19	0.3687	0.1794	0.0627	0.3892	38369.88	12955.2	731.1664	1.3837
20	0.421	0.0319	0	0.547	38938.25	14563.35	715.0679	1.3291
21	0.0119	0.4165	0.5715	0	45020.94	12109.69	776.6504	1.4062
22	0.0466	0.1694	0.7773	0.0067	42997.04	12475.15	787.5743	1.4062
23	0	0.3998	0.2686	0.3316	45367.03	12173.38	753.0088	1.3595
24	0.8032	0.0098	0.0005	0.1865	37236.32	13560.23	717.7176	1.4049
25	0.0329	0	0.6159	0.3513	42928.7	14725.35	796.8851	1.2981
26	0.5056	0.0003	0	0.494	38441.14	14419.48	715.0679	1.3472
27	0.0352	0.766	0.0418	0.157	41376.68	11879.14	738.6567	1.4062
28	0.0407	0.3944	0.1865	0.3785	44847.48	12221.38	742.5613	1.3474
29	0.5923	0.066	0	0.3417	37636.91	13958.4	717.9521	1.3792
30	0.0992	0.6287	0.0034	0.2687	41504.01	11928.74	726.0321	1.3968
31	0.1354	0.0126	0.6272	0.2248	41553.97	14255.61	796.8851	1.3382
32	0.7565	0.0003	0.2322	0.011	37444.99	13333.38	752.7839	1.4062
33	0.0067	0.0147	0.0053	0.9734	44316.32	12753.82	726.8117	1.2981
34	0.2406	0.0015	0.38	0.3778	40017.59	13979.94	768.9911	1.3306
35	0.6052	0.2998	0.004	0.091	38063.29	12632.53	729.4745	1.4062

Set	w_1	w_2	w_3	w_4	OEM	Fuel Mass	Utilization/ (Block time)	Passenger Density
36	0.2621	0.7258	0.0121	0	39967.69	12039.81	727.7391	1.4062
37	0.9446	0.0535	0.0018	0.0001	37287.83	13221.59	736.5761	1.4062
38	0.9157	0	0.0667	0.0176	37275.52	13276.58	743.8319	1.4062
39	0.1839	0.3162	0.3597	0.1403	40042.67	12242.96	762.1718	1.4062
40	0.9923	0.0047	0	0.003	37276.06	13418.7	722.9659	1.4062
41	0.5623	0.3139	0.1176	0.0062	38037.84	12646.94	732.2106	1.4062
42	0.002	0.2278	0.0097	0.7604	45992.66	12608.39	731.5576	1.3016
43	0.2451	0.011	0.2933	0.4506	40118.47	14173.88	756.1211	1.3108
44	0.4803	0.2422	0.0302	0.2472	38049.27	12641.76	728.0511	1.4062
45	0.4546	0.2123	0.0078	0.3252	37937.91	12703.52	727.4846	1.4062
46	0.3963	0.0912	0.2477	0.2648	37698.98	13163.99	756.1622	1.4024
47	0.112	0.1415	0.4848	0.2617	41546.1	12918.15	782.1182	1.365
48	0.2459	0	0.0145	0.7396	39728.33	14790.34	715.0679	1.303
49	0.6045	0.0059	0.3867	0.0029	37765.89	13298.83	763.2272	1.4062
50	0.4188	0.1814	0.067	0.3327	37868.71	12785.68	730.2064	1.4042
51	0.4102	0.0687	0.1709	0.3502	37721.02	13559.16	736.8785	1.3838
52	0.5535	0.0122	0.1728	0.2615	37188.6	13456.12	739.1604	1.4059
53	0.8347	0.133	0.0001	0.0322	37436.49	13056.45	730.646	1.4062
54	0.5616	0.0789	0.0309	0.3286	37614.04	13795.63	723.0046	1.3817
55	0.0004	0.0186	0.9475	0.0335	46824.92	13544.52	796.8851	1.3418
56	0.0422	0.5609	0.3625	0.0344	43397.63	11862.87	766.3542	1.4211
57	0.3347	0.0001	0.6348	0.0304	38774.75	13482.08	784.6366	1.4062
58	0.4379	0.0001	0.5606	0.0013	38276.79	13334.99	775.4621	1.4062
59	0.0654	0.5942	0.2612	0.0792	42023.53	11841.26	760.0369	1.4211
60	0.4001	0.2158	0.0149	0.3691	38159.24	12696.21	729.1825	1.4007
61	0.1627	0.0297	0.355	0.4527	41623.78	14327.67	776.0346	1.291
62	0.0001	0.4	0.2372	0.3627	45285.95	12210.96	748.9781	1.3518
63	0.4422	0.4512	0.0064	0.1001	38831.82	12318.31	728.4445	1.4062
64	0.6081	0.1174	0.0758	0.1987	37436.74	13058.77	735.54	1.4062
65	0.0302	0.7622	0.1212	0.0865	42135.08	11780.44	753.4676	1.4211
66	0	0.5144	0.0031	0.4825	44201.8	12139.96	734.222	1.357
67	0.5574	0.1117	0.0302	0.3007	37501.71	13317.47	729.5176	1.396
68	0.5882	0.0914	0.3203	0.0001	37720.14	13145.69	760.5986	1.4062
69	0.0006	0.0052	0.9879	0.0062	44975.87	13130.95	796.8851	1.3877
70	0.2094	0.0095	0.7634	0.0176	39058	13491.32	793.4018	1.4211

Set	w_1	w_2	w_3	w_4	OEM	Fuel Mass	Utilization/ (Block time)	Passenger Density
71	0.0581	0.4849	0.2125	0.2445	43262.31	11964.33	751.1284	1.3888
72	0.1297	0.575	0.0001	0.2953	40919.2	11987.3	726.6097	1.3976
73	0.3782	0.4645	0.1531	0.0043	39049.01	12253.68	734.0083	1.4062
74	0	0.5184	0.4477	0.0338	44682.3	11908.9	770.7272	1.4211
75	0.0001	0.9878	0	0.0121	43817.67	11768.92	734.1318	1.4211
76	0.003	0.409	0.543	0.045	44795.68	11999.31	776.1666	1.4211
77	0.2354	0.0022	0	0.7624	39874.87	14847.17	715.0679	1.2981
78	0.0085	0.0079	0.4356	0.5479	43304.69	14519.47	796.8851	1.2981
79	0.0006	0.0511	0.7425	0.2057	48529.98	14046.5	796.8851	1.294
80	0.1498	0.0178	0.0004	0.8319	39878.4	14800.82	715.0679	1.2981
81	0.032	0.6551	0.0096	0.3033	43319.39	11963.59	732.1487	1.3799
82	0.9906	0	0.0032	0.0061	36947.09	13279.07	722.8448	1.4211
83	0.3111	0.655	0.0151	0.0189	39678.05	12098.1	726.9415	1.4062
84	0.429	0.0698	0.5011	0	38191.41	13201.07	773.1662	1.4062
85	0.0602	0.0714	0.4478	0.4206	43950.06	13881.94	787.057	1.2857
86	0.0002	0.18	0.7782	0.0416	45402.92	12464.13	788.5213	1.4062
87	0.0232	0.3681	0.3557	0.253	44783.21	12006.7	765.5023	1.3986
88	0.0109	0.114	0.2348	0.6403	48694.94	13019.07	768.3219	1.2857
89	0.2501	0.388	0.1352	0.2267	39450.37	12155.04	736.9588	1.4062
90	0.0249	0.4659	0.3909	0.1184	44115.42	11902.98	770.0601	1.4211
91	0.0011	0.0019	0.997	0	42087.9	13094.97	796.8851	1.4062
92	0.0092	0.0696	0.4829	0.4382	48113.29	13439.78	787.8327	1.2981
93	0.0129	0.5555	0.1046	0.327	44036	12029.99	736.3019	1.3708
94	0.0543	0.7987	0.0003	0.1467	41044.31	11887.42	729.5942	1.4062
95	0	0	0.641	0.359	45729.69	16276.61	796.8851	1.2981
96	0.489	0.026	0.135	0.35	37935.02	13970.83	726.1814	1.3687
97	0.1027	0.631	0.0548	0.2115	40748.32	11920.6	732.8819	1.4062
98	0.458	0.0788	0.3237	0.1395	37476.49	13025.54	763.1866	1.4211
99	0.1384	0.0501	0.7969	0.0146	39310.86	13431.58	796.5039	1.4211
100	0.0008	0.083	0.9151	0.0011	44887.95	12965.04	796.8428	1.4062

D.2 Additional Untried Data for Evaluation of Surrogate Model Accuracy

The 84 sets of additional untried data for weighting factors and the actual values of four design criteria obtained by the analysis tool (VAMPzero), are listed in Table D.2.

The predicted values of four design criteria for the 84 additional untried data points of weighting factors, generated by the constructed surrogated models, are listed in Table D.3. The relative error is the difference between the predicted values and the actual values.

Table D.2: The 84 Sets of Weighting Factors and Predicted Design Criteria Values, Obtained by the Analysis Tool (VAMPzero)

Set	w_1	w_2	w_3	w_4	OEM	Fuel Mass	Utilization/ (Block time)	Passenger Density
1	0.1	0.1	0.1	0.7	42084.41	13197.05	739.95	1.2981
2	0.1	0.1	0.2	0.6	42341.09	13318.56	758.24	1.2981
3	0.1	0.1	0.3	0.5	42713.95	13500.98	772.32	1.2981
4	0.1	0.1	0.4	0.4	43005.37	13719.57	782.12	1.2953
5	0.1	0.1	0.5	0.3	42366.91	13573.82	786.28	1.3211
6	0.1	0.1	0.6	0.2	40645.46	13125.85	788.74	1.3828
7	0.1	0.1	0.7	0.1	40033.5	12977.04	790.11	1.4063
8	0.1	0.2	0.1	0.6	43927.88	12671.43	730.95	1.3084
9	0.1	0.2	0.2	0.5	43598.42	12656.58	746.36	1.3177
10	0.1	0.2	0.3	0.4	43509.83	12689.26	758.81	1.3253
11	0.1	0.2	0.4	0.3	42588.78	12561.7	771.13	1.3608
12	0.1	0.2	0.5	0.2	40981.43	12278.35	780.33	1.4211
13	0.1	0.2	0.6	0.1	40925.32	12344.49	782.6	1.4211
14	0.1	0.3	0.1	0.5	43928.19	12369.03	730.05	1.334
15	0.1	0.3	0.2	0.4	43221.94	12245.3	742.12	1.3546
16	0.1	0.3	0.3	0.3	42486.52	12216.92	757.76	1.3748
17	0.1	0.3	0.4	0.2	41583.82	12152.49	770.24	1.4063
18	0.1	0.3	0.5	0.1	41441.19	12126.11	775.59	1.4211
19	0.1	0.4	0.1	0.4	43174.65	12094.29	728.62	1.3661
20	0.1	0.4	0.2	0.3	41990.3	11968.47	744.85	1.3933
21	0.1	0.4	0.3	0.2	41147.74	11925.88	761.51	1.4211
22	0.1	0.4	0.4	0.1	41798.45	12088.73	767.56	1.4063
23	0.1	0.5	0.1	0.3	42256.88	11999.73	731.87	1.3826
24	0.1	0.5	0.2	0.2	39024.56	12896.65	715.07	1.4063
25	0.1	0.5	0.3	0.1	41559.88	11999.51	759.42	1.4063
26	0.1	0.6	0.1	0.2	40476.15	11819.08	738.77	1.4211
27	0.1	0.6	0.2	0.1	40839.1	11844.64	750.5	1.4211
28	0.1	0.7	0.1	0.1	40851.2	11921.26	738.8	1.4063
29	0.2	0.1	0.1	0.6	40426.28	13838.65	734.25	1.302
30	0.2	0.1	0.2	0.5	40243.59	13875.71	751.87	1.3103
31	0.2	0.1	0.3	0.4	39820.91	13608.73	764.65	1.338
32	0.2	0.1	0.4	0.3	39255.84	13260.78	776.65	1.3791
33	0.2	0.1	0.5	0.2	38983.05	13082.04	784.59	1.4063
34	0.2	0.1	0.6	0.1	39106.72	13104.04	786.58	1.4063
35	0.2	0.2	0.1	0.5	40827.52	12772.78	729.9	1.3406
36	0.2	0.2	0.2	0.4	40109.26	12582.87	743.24	1.3676
37	0.2	0.2	0.3	0.3	39549.13	12492	759.68	1.394
38	0.2	0.2	0.4	0.2	39421.6	12542.33	770.99	1.4063
39	0.2	0.2	0.5	0.1	39560.15	12632.38	776.52	1.4063
40	0.2	0.3	0.1	0.4	40881.05	12372.48	733.15	1.3663
41	0.2	0.3	0.2	0.3	39707.49	12183.51	745.69	1.4028
42	0.2	0.3	0.3	0.2	39816.73	12221.34	757.16	1.4063

Set	w_1	w_2	w_3	w_4	OEM	Fuel Mass	Utilization/ (Block time)	Passenger Density
43	0.2	0.3	0.4	0.1	39889.22	12319.01	765.05	1.4063
44	0.2	0.4	0.1	0.3	39801.33	12071.75	732.91	1.4063
45	0.2	0.4	0.2	0.2	39836.48	12105.08	744.44	1.4063
46	0.2	0.4	0.3	0.1	40000.09	12153.63	754.26	1.4063
47	0.2	0.5	0.1	0.2	39929.03	12047.41	733.31	1.4063
48	0.2	0.5	0.2	0.1	40009.31	12064.93	743.08	1.4063
49	0.2	0.6	0.1	0.1	40057.1	12022.29	730.83	1.4063
50	0.3	0.1	0.1	0.5	38916.96	13947.73	732.46	1.338
51	0.3	0.1	0.2	0.4	38448.22	13562.88	745.3	1.3634
52	0.3	0.1	0.3	0.3	38281.35	13243.09	761.36	1.3868
53	0.3	0.1	0.4	0.2	38260.71	13079.76	772.99	1.4063
54	0.3	0.1	0.5	0.1	38515.93	13101.09	778.1	1.4063
55	0.3	0.2	0.1	0.4	38850.98	12669.81	732.21	1.3856
56	0.3	0.2	0.2	0.3	38061.69	12431.43	747.69	1.4211
57	0.3	0.2	0.3	0.2	38512.93	12631.18	758.71	1.4063
58	0.3	0.2	0.4	0.1	38336.97	12605.89	767.34	1.4211
59	0.3	0.3	0.1	0.3	38880.9	12304.95	735.19	1.4063
60	0.3	0.3	0.2	0.2	38564.49	12217.94	743.17	1.4211
61	0.3	0.3	0.3	0.1	38979.96	12392.75	753.44	1.4063
62	0.3	0.4	0.1	0.2	39181.04	12214.6	731.38	1.4063
63	0.3	0.4	0.2	0.1	39203.72	12231.56	740.89	1.4063
64	0.3	0.5	0.1	0.1	39413.73	12155.35	730.77	1.4063
65	0.4	0.1	0.1	0.4	38316.4	13849	731.16	1.3576
66	0.4	0.1	0.2	0.3	37788.12	13306.97	748.09	1.3907
67	0.4	0.1	0.3	0.2	37520.62	12970.04	763.34	1.4211
68	0.4	0.1	0.4	0.1	38073.13	13124.42	769.75	1.4063
69	0.4	0.2	0.1	0.3	37995.52	12667.69	733.27	1.4063
70	0.4	0.2	0.2	0.2	38056.93	12691.44	745.71	1.4063
71	0.4	0.2	0.3	0.1	38170.97	12766.63	757.09	1.4063
72	0.4	0.3	0.1	0.2	38488.01	12441.14	732.61	1.4063
73	0.4	0.3	0.2	0.1	38178.45	12344.36	741.95	1.4211
74	0.4	0.4	0.1	0.1	38806.09	12326.12	731.02	1.4063
75	0.5	0.1	0.1	0.3	37559.28	13460.97	735.36	1.3904
76	0.5	0.1	0.2	0.2	37530.17	13128.12	752.63	1.4063
77	0.5	0.1	0.3	0.1	37460.73	13002.29	762.31	1.4211
78	0.5	0.2	0.1	0.2	37855.33	12753.19	735.58	1.4063
79	0.5	0.2	0.2	0.1	37894.05	12783.5	745.67	1.4063
80	0.5	0.3	0.1	0.1	38237.9	12546.39	731.61	1.4063
81	0.6	0.1	0.1	0.2	37381.97	13143.11	741.88	1.4063
82	0.6	0.1	0.2	0.1	37522.52	13131.66	752.42	1.4063
83	0.6	0.2	0.1	0.1	37754.16	12815.92	727.53	1.4063
84	0.7	0.1	0.1	0.1	37067.43	13008.91	742.96	1.4211

Table D.3: Predicted Design Criteria Values for the 84 Data Points and Relative Error(%), Generated by Surrogated Models

Set	OEM	Error	Fuel Mass	Error	Utilization/ (Block time)	Error	Passenger Density	Error
1	44121.58	4.84	13446.44	1.89	747.15	0.97	1.2969	-0.09
2	44280.13	4.58	13428.06	0.82	757.79	-0.06	1.3013	0.25
3	43544.43	1.94	13384.24	-0.86	764.34	-1.03	1.3131	1.16
4	42864.55	-0.33	13339.60	-2.77	771.67	-1.34	1.3267	2.42
5	42642.29	0.65	13296.15	-2.05	781.36	-0.63	1.3405	1.47
6	42731.12	5.13	13233.22	0.82	791.68	0.37	1.3563	-1.91
7	42436.23	6.00	13107.52	1.01	797.58	0.95	1.3798	-1.88
8	43956.08	0.06	12838.06	1.32	739.86	1.22	1.3154	0.54
9	43988.50	0.89	12709.90	0.42	747.93	0.21	1.3246	0.52
10	43326.04	-0.42	12587.12	-0.80	757.14	-0.22	1.3416	1.23
11	42768.23	0.42	12505.32	-0.45	768.78	-0.30	1.3603	-0.04
12	42566.32	3.87	12477.45	1.62	780.87	0.07	1.3780	-3.03
13	42423.25	3.66	12493.81	1.21	788.09	0.70	1.3958	-1.78
14	43362.23	-1.29	12487.26	0.96	735.96	0.81	1.3401	0.46
15	43130.25	-0.21	12298.43	0.43	744.71	0.35	1.3549	0.03
16	42520.69	0.08	12150.75	-0.54	756.77	-0.13	1.3753	0.03
17	42182.57	1.44	12090.76	-0.51	769.86	-0.05	1.3940	-0.87
18	42216.58	1.87	12142.36	0.13	778.43	0.37	1.4075	-0.95
19	42521.28	-1.51	12257.33	1.35	735.33	0.92	1.3667	0.05
20	42073.31	0.20	12057.71	0.75	746.14	0.17	1.3854	-0.57
21	41683.05	1.30	11939.92	0.12	759.36	-0.28	1.4043	-1.18
22	41848.95	0.12	11961.45	-1.05	769.17	0.21	1.4152	0.64
23	41650.02	-1.44	12071.60	0.60	737.24	0.73	1.3909	0.60
24	41221.22	5.63	11911.81	-7.64	749.59	4.83	1.4091	0.20
25	41403.36	-0.38	11879.45	-1.00	760.43	0.13	1.4189	0.90
26	41000.79	1.30	11913.42	0.80	740.33	0.21	1.4086	-0.88
27	41013.05	0.43	11844.84	0.00	751.85	0.18	1.4190	-0.14
28	40861.53	0.03	11826.21	-0.80	742.65	0.52	1.4155	0.66
29	41324.67	2.22	13708.68	-0.94	736.82	0.35	1.3192	1.33
30	41237.89	2.47	13501.86	-2.69	747.17	-0.62	1.3307	1.55
31	40584.85	1.92	13287.95	-2.36	758.11	-0.86	1.3450	0.52
32	40105.49	2.16	13113.39	-1.11	770.99	-0.73	1.3589	-1.46
33	39991.43	2.59	13001.98	-0.61	783.84	-0.10	1.3728	-2.38
34	39886.02	1.99	12954.85	-1.14	791.42	0.62	1.3907	-1.11
35	40966.72	0.34	12975.72	1.59	735.64	0.79	1.3461	0.41
36	40704.91	1.49	12711.93	1.03	745.68	0.33	1.3601	-0.54
37	40141.52	1.50	12495.74	0.03	758.47	-0.16	1.3763	-1.27
38	39865.91	1.13	12384.54	-1.26	771.80	0.10	1.3905	-1.12
39	39919.20	0.91	12413.08	-1.74	780.14	0.47	1.4022	-0.29
40	40557.23	-0.79	12512.06	1.13	735.56	0.33	1.3736	0.53
41	40087.88	0.96	12246.50	0.52	746.64	0.13	1.3894	-0.96
42	39699.55	-0.29	12088.16	-1.09	759.61	0.32	1.4036	-0.19

Set	OEM	Error	Fuel Mass	Error	Utilization/ (Block time)	Error	Passenger Density	Error
43	39831.10	-0.15	12105.36	-1.73	768.65	0.47	1.4111	0.35
44	40160.92	0.90	12203.28	1.09	736.44	0.48	1.3975	-0.63
45	39638.23	-0.50	11991.92	-0.93	748.08	0.49	1.4114	0.37
46	39697.14	-0.76	11952.29	-1.66	757.66	0.45	1.4170	0.76
47	39878.07	-0.13	11995.02	-0.43	737.57	0.58	1.4135	0.51
48	39642.97	-0.92	11894.54	-1.41	747.38	0.58	1.4192	0.92
49	39844.54	-0.53	11892.93	-1.08	737.59	0.93	1.4174	0.79
50	39545.60	1.62	13692.47	-1.83	732.58	0.02	1.3450	0.53
51	39232.20	2.04	13357.27	-1.52	744.43	-0.12	1.3600	-0.25
52	38696.10	1.08	13064.44	-1.35	758.65	-0.36	1.3738	-0.94
53	38467.08	0.54	12882.19	-1.51	773.07	0.01	1.3851	-1.51
54	38526.61	0.03	12856.13	-1.87	782.21	0.53	1.3964	-0.70
55	39123.68	0.70	12922.42	1.99	734.82	0.36	1.3752	-0.75
56	38651.65	1.55	12598.63	1.35	747.04	-0.09	1.3897	-2.21
57	38286.88	-0.59	12395.73	-1.86	760.76	0.27	1.4008	-0.38
58	38408.61	0.19	12392.89	-1.69	770.23	0.38	1.4066	-1.02
59	38928.34	0.12	12433.42	1.04	736.18	0.13	1.4004	-0.42
60	38370.94	-0.50	12182.30	-0.29	748.27	0.69	1.4117	-0.66
61	38368.71	-1.57	12135.52	-2.08	757.93	0.60	1.4145	0.59
62	38907.81	-0.70	12133.32	-0.67	736.54	0.71	1.4163	0.72
63	38525.01	-1.73	12016.91	-1.75	746.13	0.71	1.4190	0.91
64	39045.87	-0.93	11990.04	-1.36	735.17	0.60	1.4188	0.89
65	38374.27	0.15	13505.81	-2.48	732.14	0.13	1.3716	1.03
66	37918.26	0.34	13121.48	-1.39	746.03	-0.28	1.3864	-0.31
67	37598.66	0.21	12860.02	-0.85	761.20	-0.28	1.3962	-1.75
68	37735.10	-0.89	12811.47	-2.38	771.94	0.28	1.4016	-0.33
69	38004.05	0.02	12786.20	0.94	735.74	0.34	1.3998	-0.46
70	37471.11	-1.54	12497.18	-1.53	749.15	0.46	1.4100	0.27
71	37469.83	-1.84	12433.40	-2.61	759.91	0.37	1.4116	0.38
72	38039.89	-1.16	12359.36	-0.66	736.81	0.57	1.4172	0.78
73	37609.04	-1.49	12232.99	-0.90	747.37	0.73	1.4183	-0.19
74	38313.51	-1.27	12155.45	-1.38	735.24	0.58	1.4197	0.96
75	37546.53	-0.03	13256.72	-1.52	733.85	-0.21	1.3955	0.37
76	37097.22	-1.15	12921.64	-1.57	749.09	-0.47	1.4063	0.00
77	37158.98	-0.81	12821.03	-1.39	761.62	-0.09	1.4083	-0.90
78	37330.91	-1.39	12675.06	-0.61	737.40	0.25	1.4161	0.70
79	36951.65	-2.49	12534.73	-1.95	749.75	0.55	1.4172	0.78
80	37602.16	-1.66	12397.90	-1.18	737.09	0.75	1.4202	0.99
81	36944.18	-1.17	13053.21	-0.68	736.68	-0.70	1.4130	0.48
82	36716.17	-2.15	12884.92	-1.88	751.34	-0.14	1.4155	0.66
83	36973.26	-2.07	12697.02	-0.93	739.39	1.63	1.4202	0.99
84	36594.99	-1.27	13003.29	-0.04	740.22	-0.37	1.4197	-0.10

D.3 Typical Weighting Scenarios for Business Aircraft Evaluation

In the business aircraft evaluation problem using ELECTRE I, 84 sets of weighting factors generated from eleven levels of experimental design and evaluation results are summarized in Table D.4, where D represents the alternative is dominated, and N represents the alternative is non-dominated.

Table D.4: The 84 Sets of Weighting Factors for Business Aircraft Evaluation, D: Dominated, N: Non-dominated

Set	w_1	w_2	w_3	w_4	w_5	w_6	w_7	A_1	A_2	A_3	A_4
1	0.4	0.1	0.1	0.1	0.1	0.1	0.1	D	D	N	D
2	0.3	0.1	0.1	0.1	0.1	0.1	0.2	D	D	N	D
3	0.3	0.1	0.1	0.1	0.1	0.2	0.1	D	D	N	D
4	0.3	0.1	0.1	0.1	0.2	0.1	0.1	N	D	N	D
5	0.3	0.1	0.1	0.2	0.1	0.1	0.1	N	D	N	D
6	0.3	0.1	0.2	0.1	0.1	0.1	0.1	N	D	N	D
7	0.3	0.2	0.1	0.1	0.1	0.1	0.1	D	D	N	D
8	0.2	0.1	0.1	0.1	0.1	0.1	0.3	D	D	N	D
9	0.2	0.1	0.1	0.1	0.1	0.2	0.2	D	D	N	D
10	0.2	0.1	0.1	0.1	0.1	0.3	0.1	D	N	N	D
11	0.2	0.1	0.1	0.1	0.2	0.1	0.2	N	D	N	D
12	0.2	0.1	0.1	0.1	0.2	0.2	0.1	N	D	N	D
13	0.2	0.1	0.1	0.1	0.3	0.1	0.1	N	D	D	D
14	0.2	0.1	0.1	0.2	0.1	0.1	0.2	D	D	N	D
15	0.2	0.1	0.1	0.2	0.1	0.2	0.1	D	N	N	D
16	0.2	0.1	0.1	0.2	0.2	0.1	0.1	N	D	D	N
17	0.2	0.1	0.1	0.3	0.1	0.1	0.1	N	N	N	N
18	0.2	0.1	0.2	0.1	0.1	0.1	0.2	D	D	N	N
19	0.2	0.1	0.2	0.1	0.1	0.2	0.1	N	D	N	N
20	0.2	0.1	0.2	0.1	0.2	0.1	0.1	N	D	D	N
21	0.2	0.1	0.2	0.2	0.1	0.1	0.1	N	D	N	N
22	0.2	0.1	0.3	0.1	0.1	0.1	0.1	N	D	D	N
23	0.2	0.2	0.1	0.1	0.1	0.1	0.2	D	D	N	D
24	0.2	0.2	0.1	0.1	0.1	0.2	0.1	D	N	N	D
25	0.2	0.2	0.1	0.1	0.2	0.1	0.1	N	D	D	D
26	0.2	0.2	0.1	0.2	0.1	0.1	0.1	D	N	N	D
27	0.2	0.2	0.2	0.1	0.1	0.1	0.1	N	D	N	N
28	0.2	0.3	0.1	0.1	0.1	0.1	0.1	D	N	N	D
29	0.1	0.1	0.1	0.1	0.1	0.1	0.4	D	D	N	N
30	0.1	0.1	0.1	0.1	0.1	0.2	0.3	D	D	N	N
31	0.1	0.1	0.1	0.1	0.1	0.3	0.2	D	N	N	N
32	0.1	0.1	0.1	0.1	0.1	0.4	0.1	D	N	N	N
33	0.1	0.1	0.1	0.1	0.2	0.1	0.3	D	D	N	N
34	0.1	0.1	0.1	0.1	0.2	0.2	0.2	N	D	N	N
35	0.1	0.1	0.1	0.1	0.2	0.3	0.1	N	N	N	N
36	0.1	0.1	0.1	0.1	0.3	0.1	0.2	N	D	D	N
37	0.1	0.1	0.1	0.1	0.3	0.2	0.1	N	D	D	D
38	0.1	0.1	0.1	0.1	0.4	0.1	0.1	N	D	D	D
39	0.1	0.1	0.1	0.2	0.1	0.1	0.3	D	D	N	N
40	0.1	0.1	0.1	0.2	0.1	0.2	0.2	D	N	N	N

Set	w_1	w_2	w_3	w_4	w_5	w_6	w_7	A_1	A_2	A_3	A_4
41	0.1	0.1	0.1	0.2	0.1	0.3	0.1	D	N	N	N
42	0.1	0.1	0.1	0.2	0.2	0.1	0.2	N	D	N	N
43	0.1	0.1	0.1	0.2	0.2	0.2	0.1	N	N	D	N
44	0.1	0.1	0.1	0.2	0.3	0.1	0.1	N	D	D	D
45	0.1	0.1	0.1	0.3	0.1	0.1	0.2	N	N	N	N
46	0.1	0.1	0.1	0.3	0.1	0.2	0.1	N	N	N	N
47	0.1	0.1	0.1	0.3	0.2	0.1	0.1	N	N	D	N
48	0.1	0.1	0.1	0.4	0.1	0.1	0.1	N	N	N	N
49	0.1	0.1	0.2	0.1	0.1	0.1	0.3	D	D	N	N
50	0.1	0.1	0.2	0.1	0.1	0.2	0.2	D	D	N	N
51	0.1	0.1	0.2	0.1	0.1	0.3	0.1	N	N	N	N
52	0.1	0.1	0.2	0.1	0.2	0.1	0.2	N	D	D	N
53	0.1	0.1	0.2	0.1	0.2	0.2	0.1	N	D	D	N
54	0.1	0.1	0.2	0.1	0.3	0.1	0.1	N	D	D	D
55	0.1	0.1	0.2	0.2	0.1	0.1	0.2	N	D	D	N
56	0.1	0.1	0.2	0.2	0.1	0.2	0.1	N	N	D	N
57	0.1	0.1	0.2	0.2	0.2	0.1	0.1	N	D	D	N
58	0.1	0.1	0.2	0.3	0.1	0.1	0.1	N	N	D	N
59	0.1	0.1	0.3	0.1	0.1	0.1	0.2	N	D	D	N
60	0.1	0.1	0.3	0.1	0.1	0.2	0.1	N	D	D	N
61	0.1	0.1	0.3	0.1	0.2	0.1	0.1	N	D	D	N
62	0.1	0.1	0.3	0.2	0.1	0.1	0.1	N	D	D	N
63	0.1	0.1	0.4	0.1	0.1	0.1	0.1	N	D	D	N
64	0.1	0.2	0.1	0.1	0.1	0.1	0.3	D	D	N	N
65	0.1	0.2	0.1	0.1	0.1	0.2	0.2	D	N	N	N
66	0.1	0.2	0.1	0.1	0.1	0.3	0.1	D	N	N	N
67	0.1	0.2	0.1	0.1	0.2	0.1	0.2	N	D	N	N
68	0.1	0.2	0.1	0.1	0.2	0.2	0.1	N	N	D	N
69	0.1	0.2	0.1	0.1	0.3	0.1	0.1	N	D	D	D
70	0.1	0.2	0.1	0.2	0.1	0.1	0.2	D	N	N	N
71	0.1	0.2	0.1	0.2	0.1	0.2	0.1	D	N	N	N
72	0.1	0.2	0.1	0.2	0.2	0.1	0.1	N	N	D	N
73	0.1	0.2	0.1	0.3	0.1	0.1	0.1	N	N	N	N
74	0.1	0.2	0.2	0.1	0.1	0.1	0.2	D	D	D	N
75	0.1	0.2	0.2	0.1	0.1	0.2	0.1	N	N	D	N
76	0.1	0.2	0.2	0.1	0.2	0.1	0.1	N	D	D	N
77	0.1	0.2	0.2	0.2	0.1	0.1	0.1	N	N	D	N
78	0.1	0.2	0.3	0.1	0.1	0.1	0.1	N	D	D	N
79	0.1	0.3	0.1	0.1	0.1	0.1	0.2	D	N	N	N
80	0.1	0.3	0.1	0.1	0.1	0.2	0.1	D	N	N	N
81	0.1	0.3	0.1	0.1	0.2	0.1	0.1	N	N	D	N
82	0.1	0.3	0.1	0.2	0.1	0.1	0.1	D	N	N	N
83	0.1	0.3	0.2	0.1	0.1	0.1	0.1	N	N	D	N
84	0.1	0.4	0.1	0.1	0.1	0.1	0.1	D	N	N	N

Declaration

I herewith declare that I have produced this paper without the prohibited assistance of third parties and without making use of aids other than those specified; notions taken over directly or indirectly from other sources have been identified as such. This paper has not previously been presented in identical or similar form to any other German or foreign examination board.

The thesis work was conducted from September 2008 to 2012 under the supervision of Prof. Dr.-Ing. Volker Gollnick at the Institute of Air Transportation Systems, German Aerospace Center (DLR).

Hamburg,